The Wonders of Haga's Theorem

Submitted by

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A project presented to the Singapore Mathematics Project Festival 2020

1. Introduction

Our project will be about Haga's Theorem. Prior to research, all of us felt that the Haga's Theorem was only limited to folding paper and we could not grasp the concept. However, after the help from our teacher and some research, we got to understand the theorem better and we can now apply it in our daily lives. After this project, we aim to find a way to explain Haga's theorem simply so that more people would be able to understand it.

2. Background information

Origami is an interesting activity to participate in. It involves the precise folding of a piece of paper to form a beautiful artwork but few people actually know the origins of this art form. The word origami originates from two japanese words, 'ori' which means fold and 'kami' which means paper. In the sixth century it is said that buddhist monks brought paper to Japan. This is a significant detail as it is also the time when the very first Japanese origami dates back to.

Due to the high price of paper, origami was strictly used for religious ceremonial purposes only. As time progressed so did the origami scene in Japan and in 1979, the first known origami book was published in Japan. However, the modern interest in origami is actually credited to the design made in 1954 by Akira Yoshizawa entitled "Yoshizawa-Randlett system". The design is a notation which indicates how to fold origami models and is now used internationally.

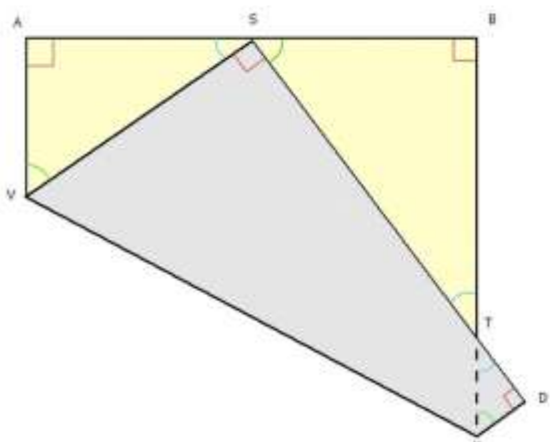
A man that has contributed to the development of origami over the years is Kazuo Haga. Kazuo Haga was born in 1934 and is a Japanese retired professor of biology from the University of Tsukuba. He founded The Haga's Theorem.

3. Haga's Theorem

Haga's theorem makes use of simple concepts such as the property of angles, Pythagoras' theorem and similar triangles to show how an origami paper can be folded into thirds, fourths, fifths, and so on without the use of a ruler. According to Haga's theorem,

$$\frac{y}{2} = \frac{1/N}{1 + 1/N} = \frac{1}{N+1}.$$

Making use of Haga's theorem, starting from ½ of the paper, one would be able to find ⅓ of the paper and knowing ⅓ of the paper one would be able to find ¼ of the paper and so on.



As all the corresponding angles of triangles VAS, SBT and UDT are the same, it proves that they are similar triangles. Therefore, the ratio of the corresponding sides of the triangles should be constant throughout.

Now, we let each side of the square be represented by one unit and as S is the middle point of AB, AS and SB are ½ each. Next, we let the length of AV to be x, which also means that the length of SV would be 1-x.

Thus, by using Pythagoras' theorem, we can infer that

$$x^2 + \left(\frac{1}{2}\right)^2 = (1 - x)^2.$$

Which, when solved, gives us x=⅜. Using this, we can calculate the length of BT, which we will call y. Since the triangles are similar, we know that the ratios of sides of the triangles are the same. So,

$$\frac{y}{\frac{1}{2}} = \frac{\frac{1}{2}}{x}.$$

Since we already know that x=3/8, we can find the y, which is 2/3. From this, we can get ⅓ by folding y in half.

Therefore, this proves that

$$\frac{y}{2} = \frac{1/N}{1 + 1/N} = \frac{1}{N+1}.$$

4. Conclusion

In conclusion, Haga's Theorem enables us to find out how to fold origami paper into equal parts easily. With Haga's Theorem, we would be able to fold origami into odd parts like thirds and fifths which would have otherwise been difficult to do without the theorem. It also shows us how the things that we have learnt can be applied together.

5. Bibliography

We would like to express our special thanks of gratitude to our teacher-in-charge, Mr Diva, for his guidance which helped us in completing this project. We would also like to thank our families and friends for their unwavering support. Lastly, we would like to thank each other for our persistence and teamwork.

6. Citations

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