# THE MATH BEHIND PARALLEL PARKING

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## ABSTRACT

This essay mainly covers the topic of Trigonometry and a little bit of Euclidean geometry. We will be covering the topics in depth and describing to others how these topics can be seen in our daily lives, mainly focusing on parallel parking to help improve the lives of people.

## ACKNOWLEDGEMENT

We would like to thank Mr Wong and Mr Diva for guiding us through this year-long project. They have given us many constructive advice and helped us through our ups and downs as this has not been an easy journey. They have been our inspiration, encouraging us. They also have been very supportive and would always tell us to not give up on the project, especially during the times where we have hit a dead end in our project. There were countless times where we did not understand the materials, but they patiently explained to us part by part.

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## INTRODUCTION

Parallel parking is not that easy, sometimes, you have to try over and over again to get your car is a good position. Parallel parking is considered a difficult skill for new drivers to learn. This got us thinking, what if we could make it simpler and help the people whom have just started to drive to park their car in a parallel lot by coming up with a formula? It is simpler if there was another empty lot in front or behind the parallel parking spot, but our formula allows the driver to be able to park in a parallel parking lot easily even with the front and back parking lots filled.

Professor Blackburn had done something similar and made a formula to help drivers determine whether the parking lot is big enough for their cars, but our formula focuses on a different aspect of parallel parking and it is much simpler too.

We spent a lot of time wondering which aspect of Math we should focus on when coming up with the formula. We went back and forth researching on topics such as Euclidean geometry and the Ackerman Steering Geometry, reading up on several things out of our syllabus, wondering which one of these topics could best help us formulate a formula to solve the problem of parallel parking. Finally, we decided on settling mainly on trigonometry as after seeing how Professor Blackburn mainly focused on one of the properties of right-angled triangles, the Pythagorean Theorem, we decided to try something similar and use trigonometry.

Hence, we used trigonometry to find a formula for parallel parking, so that drivers, especially beginner drivers who are not so used to parallel parking yet, can use this formula and get their car in a good spot in one try.

## LITERATURE REVIEW

People believe that parallel parking should not be included in driving exams as they feel like it is not a necessity.

In the past, parallel parking was a necessity in a driving test, and it was something many struggle with. Many failed their tests due to their struggle in parallel parking. Fast forward to a few years later, many places have dropped the requirement to parallel park. One example includes in the United States, 16 states did not require the successful demonstration of parallel parking in May 2019.

However, there would be times where parallel parking is required. Well, Professor Blackburn came up with a formula on how to calculate the extra space needed to parallel park better. He also explained the Mathematics behind it and this helped many people.

## METHODOLOGY

Firstly, we did research on parallel parking and why so many people found it difficult. According to users on Quora, people find parallel parking difficult due to people not knowing the size of the car, and the space between their car and the car behind them. This instills fear as they do not want to crash their car. Another reason is due to pressure. When parallel parking, it takes a lot more time than perpendicular parking and you may be blocking traffic.

We used Professor Simon R. Blackburn's essay on The Geometry of Perfect Parking to help guide us along. He explained how much extra space a car needed to parallel park and came up with a formula. His explanation of the use of the Pythagorean Theorem in the Mathematics behind parallel parking, gave us an idea.

We sketched out a scenario where there were 3 cars and the car in the center was the car that was going to be parked. One example in which you can see in Fig 3.1. We sketched out a few key diagrams of where the car parking would move. Using the length of the space between the 2 cars, and the width of the car parked behind, we managed to form a right-angled triangle. We could then use trigonometry to help us come up with a formula:

$$x = \frac{\tan^{-1}\left(\frac{z}{y}\right)}{2}$$

Let $x$ be the first turning angle, $z$ be the length of space between the 2 cars and $y$ be the width of the car parked behind.

Using this formula, we tried several times to make sure that this formula was correct. According to the website "qcode.us", the minimum size of a standard parking space is 9 feet wide and 18 feet long. This is approximately 274.32cm and 548.64cm respectively. The maximum turning angle is 65 degrees and the average is 45 degrees, according to Quora. Using these measurements we used them to find the turning angle. After calculations and taking the wheel's width into consideration, we came up with the formula.

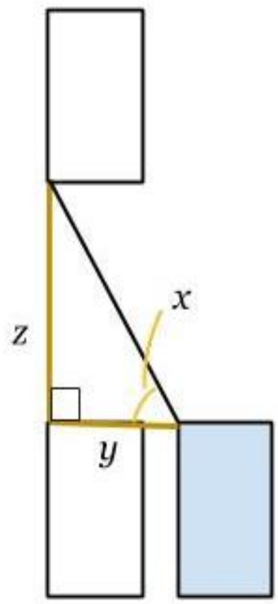



**Fig 3.1**

## ANALYSIS

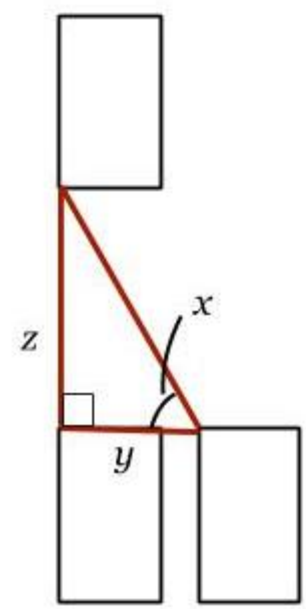
Here would be a quick analysis of the formula that was stated in the methodology as follows:

$$x = \frac{\tan^{-1}\left(\frac{z}{y}\right)}{2}$$

As well as an in-depth breakdown of the formula as well as how we came up with this formula. In this formula, $x$ is equivalent to the steering angle of the car. Please do note that this angle only applies at the start when the car is just starting to turn into the parallel parking lot and it does not apply when the car is trying to straighten into the lot. $x$ also happens to be the variable we are trying to calculate. Secondly, $z$ is equivalent to the width of the parked car (this car will be behind the car that is being parked into the lot ) as well as the gap between the two cars. For a clearer understanding, please refer to the diagram in Fig. 3.1. Lastly, $y$ refers to the length of the parking lot your car is trying to park into.

This formula uses the concept of basic trigonometry and is actually very simple considering you have access to the information required by the formula. Referring to the diagram in Fig. 3.1, we can observe that it is possible to form a right-angled triangle with the length of both $z$ and $y$, where the angle $x$, when divided by 2 is actually the turning angle of the car. By using Trigonometry, we are able to easily obtain the turning angle by using the law of sin, cos and tan and following the TOACAHSOH rule. As we have already obtained the lengths of both the opposite and adjacent lines of the right-angled triangle, we are then able to calculate the turning angle of the car by using the inverse tangent function, hence giving us the turning angle of the car.

During the process of coming up with the formula, we drew this sketch of 3 cars,2 already parked and 1 in the position that is parallel to one of the parked cars, getting ready to park. It was at this moment we realised that a right-angled triangle could be formed within these three cars.( Fig 4.1 )




**<u>Fig 4.1</u>**

The angle, *x,* can be seen as the steering angle as we would want the car to drive along the line, which can be seen as the hypotenuse of the right-angle triangle, in order to suitably get into that parallel parking lot. This, as we discovered a little later, was not true as actually we did not want the car to be "driving along" the hypotenuse, but instead we wanted the hypotenuse to fall in the middle of the car in order to nicely fit into the lot. The difference can be seen in Fig 4.2 and Fig 4.3, Fig 4.2 being if the car were to have driven "along the hypotenuse" and Fig 4.3 being if the hypotenuse fell in the middle of the car. Of course this is done with the assumption that the kerb is low or that there is nothing blocking the car from being able to drive slightly out of the space for parking.

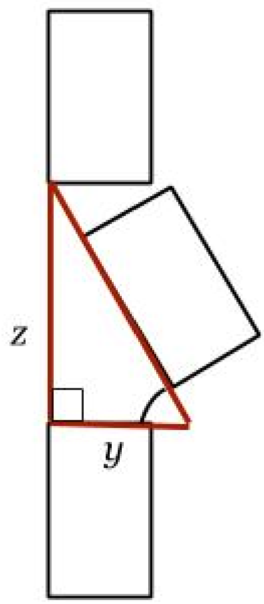



**Fig 4.2**

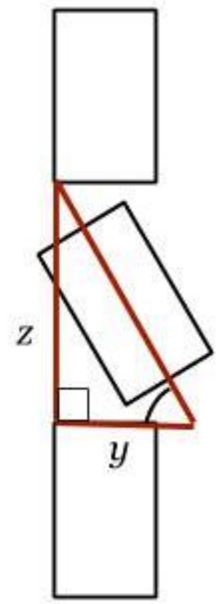



**Fig 4.3**

Thus we decided to divide our final answer by 2 as the angle would be sharper and if the car were to drive along the sharper angle, it would be the same as having the middle of the car "fall" on the hypotenuse of the right-angled triangle that was previously made before the final answer of the formula that provide us with the steering angle was divided by 2.

## CONCLUSION

In conclusion, parallel parking is actually pretty simple and can be easily overcome. Of course one can always choose to park in a perpendicular parking lot instead of a parallel lot, but we made this formula to help people parallel park safely without crashing into another car as well as to show that math is applicable into many aspects of our life, even in areas where you may never expect.

With just simple knowledge of Euclidean Geometry and low-level Trigonometry, it is possible to make a formula that can calculate what the steering angle of a car should be just to get into a certain parallel lot under certain conditions.

We went through a lot of trial and error trying to come up with this formula, wondering what variables to focus on as well as what would be a good formula that will actually be useful for drivers, so we really hope you enjoy our research as well as our take on trigonometry and parallel parking and hopefully others' will be able to gain inspiration from our project.

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