Selfie Syndrome

Submitted by S3 PLMGS(S) Students

Angela Tan Jia Yi

Naomi Faustina Limarno

Shuan Ye Ru Nicole

Wong Min Hui

Paya Lebar Methodist Girls' School (Secondary)

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Abstract

Within the past century, the act of taking selfies – a self-portrait picture, has been becoming increasingly popular. Selfie taking involves holding a phone or a digital camera right in front of you while using a hand or selfie stick for support, and it is an easy skill that most people can learn. On the contrary, taking a group selfie, also known as a 'groupie', can be a little more challenging.

The aim of our project is to find the relationship between the angle of tilt of the arm and the area of the scene that is captured in the image to fit in the maximum number of people. In order to do this, we used trigonometric ratios, and the concept of field of view and angle of view.

Firstly, we used trigonometric ratios to find the area of the field of view in terms of x in an ideal situation. As the ideal situation is unlikely to happen in reality, we used the 2 most common scenarios, namely when $\theta < \frac{A_v}{2}$, and when $\theta > \frac{A_v}{2}$, where θ is the angle of tilt of the arm from the horizontal and A_v is the vertical length of the field of view. We then managed to derive an equation that works for both of the common scenarios, but not for the ideal situation. As we were only looking at the angle of tilt of the hand, we focused on the vertical angle of view.

To further this project, we recommend finding the relationship between the area of the field of view and the horizontal angle of view.

Acknowledgement

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They are also very grateful to the project supervisor, Ms Kok Lai Fong, for her guidance and time.

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Chapter 1 - Introduction

1.1 Objective

The aim of our project is to find the relationship between the angle of tilt of the arm and the area of the scene that is captured in the image to fit in the maximum number of people when taking a 'groupie' i.e. a group selfie. This would then allow people to know the ideal angle of tilt the hand needs to satisfy to fit the maximum number of people in the 'groupie'.

1.2 Problem

Like most photographers, 'groupie'- takers may have difficulty fitting all the people that they want in their picture. It takes them time and effort in order to direct each person into a position so that they can be seen in the picture, as well as position their camera.

1.3 Background

Selfies are self-portrait photographs, which, much like other types of photographs, are often taken to commemorate special events such as birthdays or anniversaries. The word 'selfie', however, was not commonly used until several years ago.

Taking a picture of yourself was considerably rarer in the past, as it was more difficult, between longer exposure times or the need to use a mirror. With the advent of faster and front-facing cameras on phones, selfies experienced a surge in popularity and selfie-taking is even a hobby, or in some cases, an obsession.

Chapter 2 - Literature Review

2.1 Overview

We would be looking at the mathematics behind photography so as to help us devise the best way to take a groupie.

2.2 Trigonometry

Trigonometry is a branch of mathematics that studies relationships between lengths and angles within triangles. The three trigonometric functions that we are applying are sine, cosine and tangent, which are relationships between the lengths of the different sides of the triangle.

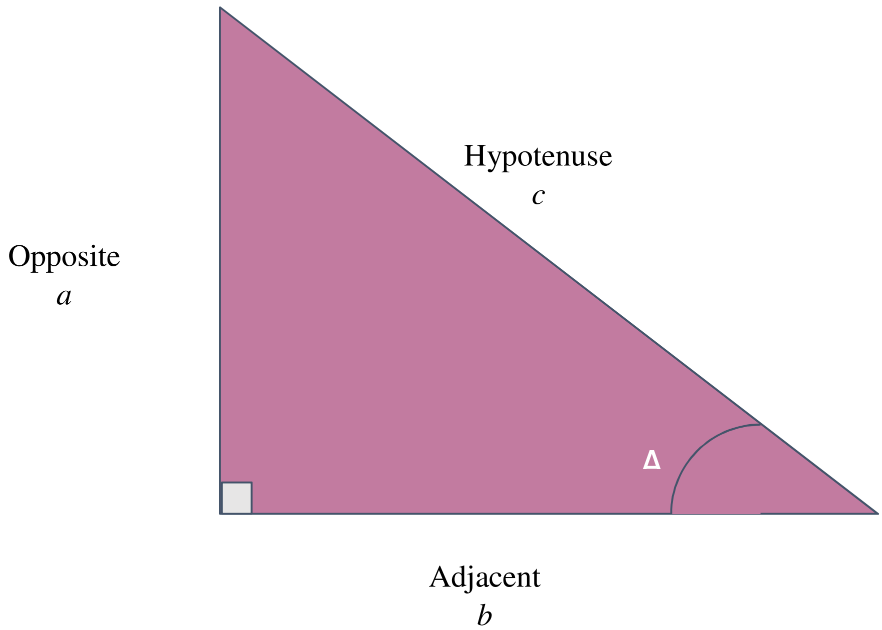

Figure 2.1 A right-angled triangle

The sine of an angle is the ratio of the side opposite the angle to the hypotenuse of the triangle. In this case:

$$\sin A = \frac{Opposite}{Hypotenuse} = \frac{a}{c}$$

The cosine of an angle is the ratio of the side adjacent to the angle to the hypotenuse of the triangle. In this case:

$$\cos A = \frac{Adjacent}{Hypotenuse} = \frac{b}{c}$$

The tangent of an angle is the ratio of the side opposite the angle to the side adjacent to the angle. In this case:

$$\tan A = \frac{Opposite}{Adjacent} = \frac{a}{b}$$

An interesting point to note would be that the sine and cosine of an angle are always < 1, because the hypotenuse is the longest side of the triangle. The tangent of an angle, however, can be > 1 as there is no strict rule about whether the side opposite or the side adjacent to the angle is longer.

2.3 Field of View and Angle of Tilt

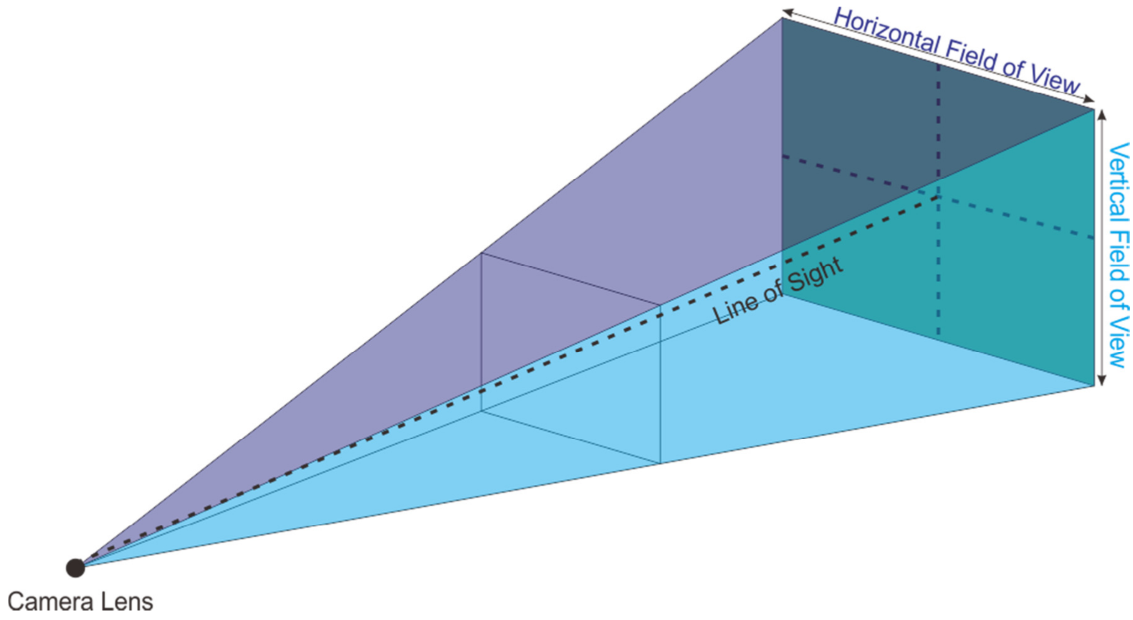

Figure 2.2 Area of View

The field of view is the maximum extent a camera can capture a landscape. The angle of view is the angular extent of the field of view. Information for the angle of view of a camera can normally be found in the specs for the camera or phone.

The angle of tilt is the angle between the arm and the horizontal from 0° to 90°. Of course, it can go beyond this number, but we will assume the 90° is the maximum.

Chapter 3 - Methodology

3.1 Overview

We would be exploring the relationship between the angle of tilt of the arm and the area captured in the picture, so as to maximise the number of people that can be fit in the picture. This would allow 'groupies' to be taken easily.

3.2 Calculating the Area of Field of View

In order to find the area of the scene that could be captured in the image, we need to find the vertical and horizontal length of the area. We would be using trigonometry to calculate this area.

The vertical length captured is l_v and the horizontal length captured is l_h. The angle of tilt of the arm is θ, the vertical angle of view is A_v, and the horizontal angle of view is A_h. x is the length of the person's arm, including a selfie stick should one be used (refer to Figure 3.1).

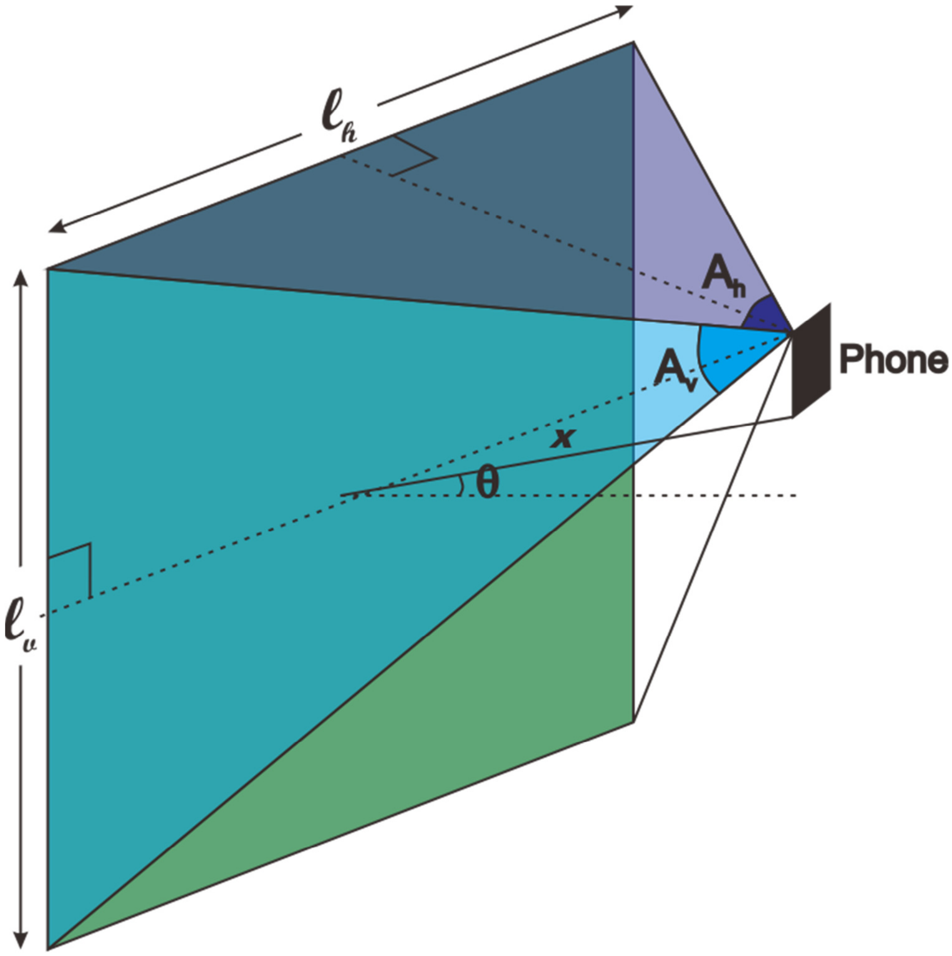

Figure 3.1 Area captured in photograph

There are 3 different scenarios, depending on what the angle of tilt of the person's arm is.

The ideal scenario occurs when $\theta = 0°$, scenario 1 occurs when $\theta < \frac{A_v}{2}$, and scenario 2 occurs when $\theta > \frac{A_v}{2}$. If $\theta = \frac{A_v}{2}$, then the formulas for both scenario 1 and 2 can be used.

3.3 Ideal Scenario

First, we find the vertical length l_v in terms of x.

$$\tan\frac{A_v}{2} = \frac{\frac{l_v}{2}}{x}$$

$$l_v = 2x\tan\frac{A_v}{2}$$

Then, we find the horizontal length l_h in terms of x.

$$\tan\frac{A_h}{2} = \frac{\frac{l_h}{2}}{x}$$

$$l_h = 2x\tan\frac{A_h}{2}$$

Multiplying the two above equations together, we get the area.

$$\begin{aligned} Area &= l_v \times l_h \\ &= (2x\tan\frac{A_v}{2})(2x\tan\frac{A_h}{2}) \\ &= 4x^2(\tan\frac{A_v}{2})(\tan\frac{A_h}{2}) \end{aligned}$$

Therefore, the equation to find the area of view when $\theta = 0°$ is $4x^2(\tan\frac{A_v}{2})(\tan\frac{A_h}{2})$.

3.4 Scenario 1

The upper length of angle of view is b, and the lower length of view is c.

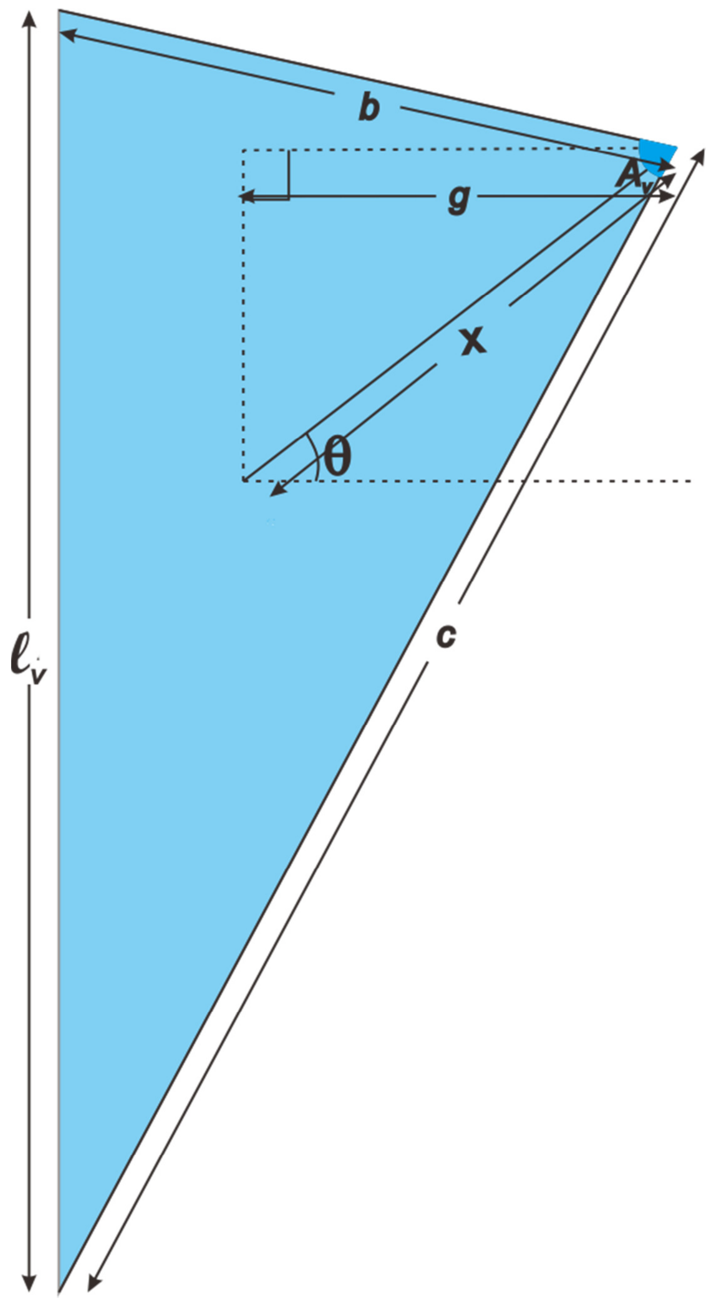

Figure 3.2 Scenario 1- Vertical length of view

We equated 2 area of triangle formulas with the intention of finding A_v in terms of x.

$$\frac{1}{2}l_v g = \frac{1}{2}bc\sin A_v$$

In order to find l_v, we need to find the length of b, c, and g.

$$\cos\theta = \frac{g}{x} \text{(Alternate ∡, parallel lines)}$$

$$g = x\cos\theta \qquad (3.1)$$

$$\cos(\theta + \frac{A_v}{2}) = \frac{g}{c}$$

$$c\cos(\theta + \frac{A_v}{2}) = g$$

Substitute equation 3.1 into the above equation.

$$c\cos(\theta + \frac{A_v}{2}) = x\cos\theta$$

$$c = \frac{x\cos\theta}{\cos(\theta + \frac{A_v}{2})} \qquad (3.2)$$

Hence,

$$\cos(\frac{A_v}{2} - \theta) = \frac{g}{b}$$

$$b\cos(\frac{A_v}{2} - \theta) = g$$

Substitute equation 3.1 into the above equation.

$$b\cos(\frac{A_v}{2} - \theta) = x\cos\theta$$

$$b = \frac{x\cos\theta}{\cos(\frac{A_v}{2} - \theta)} \qquad (3.3)$$

Since we have found the value of b, c, and g, (refer to the equations 3.1, 3.2 and 3.3), we can hence find the value of l_v.

Substitute the equations 3.1, 3.2 and 3.3 into the equation

$$\frac{1}{2}l_v g = \frac{1}{2}bc\sin A_v$$

$$\frac{1}{2}l_v(x\cos\theta) = \frac{1}{2}\left[\frac{x\cos\theta}{\cos(\frac{A_v}{2}-\theta)}\right]\left[\frac{x\cos\theta}{\cos(\frac{A_v}{2}+\theta)}\right](\sin A_v)$$

$$l_v x\cos\theta = \frac{x\cos^2\theta\sin A_v}{\cos(\frac{A_v}{2}-\theta)\cos(\frac{A_v}{2}+\theta)}$$

$$l_v = \frac{x\cos\theta\sin A_v}{\cos(\frac{A_v}{2}-\theta)\cos(\frac{A_v}{2}+\theta)} \qquad (3.4)$$

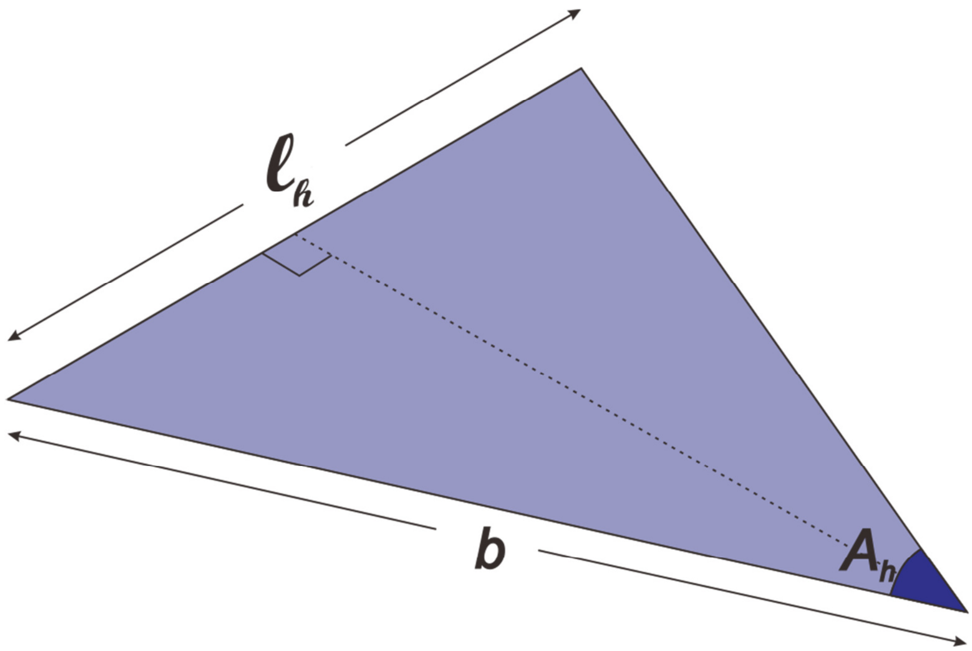

Figure 3.3 Scenario 1 - Horizontal length of view

$$\sin\frac{A_h}{2} = \frac{\frac{l_h}{2}}{b}$$

$$2b\sin\frac{A_h}{2} = l_h$$

Substitute equation 3.3 into the equation above.

$$l_h = 2[\frac{x\cos\theta}{\cos(\frac{A_v}{2}-\theta)}][\sin\frac{A_h}{2}] \quad (3.5)$$

Multiplying equation 3.4 and 3.5 together, we get the area.

$$Area = l_v \times l_h$$

$$= [\frac{x\cos\theta\sin A_v}{\cos(\frac{A_v}{2}-\theta)\cos(\frac{A_v}{2}+\theta)}][\frac{2x\cos\theta\sin\frac{A_h}{2}}{\cos(\frac{A_v}{2}-\theta)}]$$

$$= \frac{2x^2(\cos\theta)^2\sin A_v\sin\frac{A_h}{2}}{[\cos(\frac{A_v}{2}-\theta)]^2\cos(\frac{A_v}{2}+\theta)}$$

Therefore, the equation to find the area of view when $\theta < \frac{A_v}{2}$ is $\frac{2x^2(\cos\theta)^2\sin A_v\sin\frac{A_h}{2}}{[\cos(\frac{A_v}{2}-\theta)]^2\cos(\frac{A_v}{2}+\theta)}$.

3.5 Scenario 2

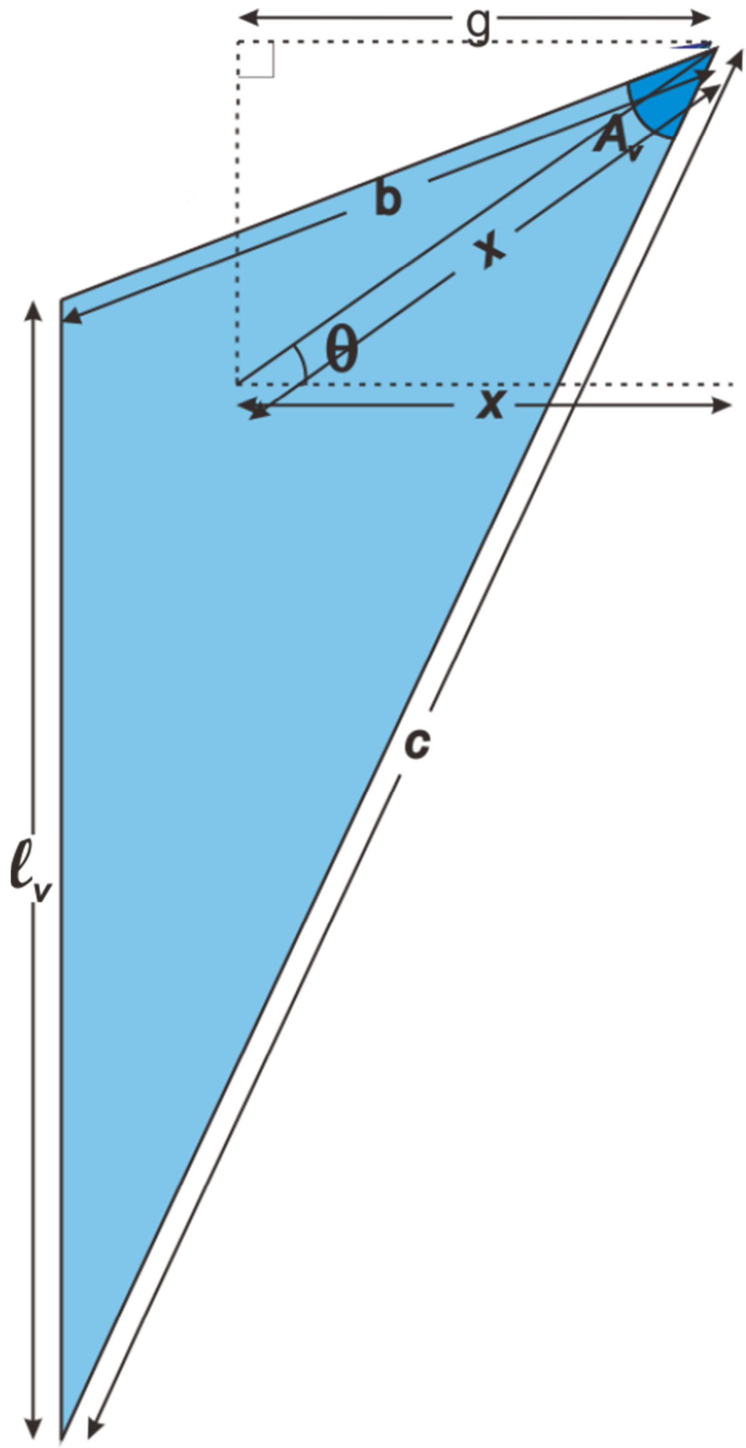

Figure 3.4 Scenario 2 - Vertical length of view

$$\cos\theta = \frac{g}{x}$$

$$g = x\cos\theta \tag{3.6}$$

$$\cos(\theta + \frac{A_v}{2}) = \frac{g}{c}$$

$$c\cos(\theta + \frac{A_v}{2}) = g$$

Sub equation 3.6 into the above equation.

$$c\cos(\theta + \frac{A_v}{2}) = x\cos\theta$$

$$c = \frac{x\cos\theta}{\cos(\theta + \frac{A_v}{2})} \tag{3.7}$$

$$\cos(\theta - \frac{A_v}{2}) = \frac{g}{b}$$

$$b\cos(\theta - \frac{A_v}{2}) = g$$

Sub equation 3.6 into the above equation.

$$b\cos(\theta - \frac{A_v}{2}) = x\cos\theta$$

$$b = \frac{x\cos\theta}{\cos(\theta - \frac{A_v}{2})} \tag{3.8}$$

Sub equation 3.6, 3.7 and 3.8 into the equation

$$\frac{1}{2}l_v g = \frac{1}{2}bc\sin A_v$$

$$\frac{1}{2}l_v(x\cos\theta) = \frac{1}{2}\left[\frac{x\cos\theta}{\cos(\theta - \frac{A_v}{2})}\right]\left[\frac{x\cos\theta}{\cos(\frac{A_v}{2} + \theta)}\right](\sin A_v)$$

$$l_v x\cos\theta = \frac{x\cos^2\theta\sin A_v}{\cos(\theta - \frac{A_v}{2})\cos(\frac{A_v}{2} + \theta)}$$

$$l_v = \frac{x\cos\theta\sin A_v}{\cos(\theta - \frac{A_v}{2})\cos(\frac{A_v}{2} + \theta)} \tag{3.9}$$

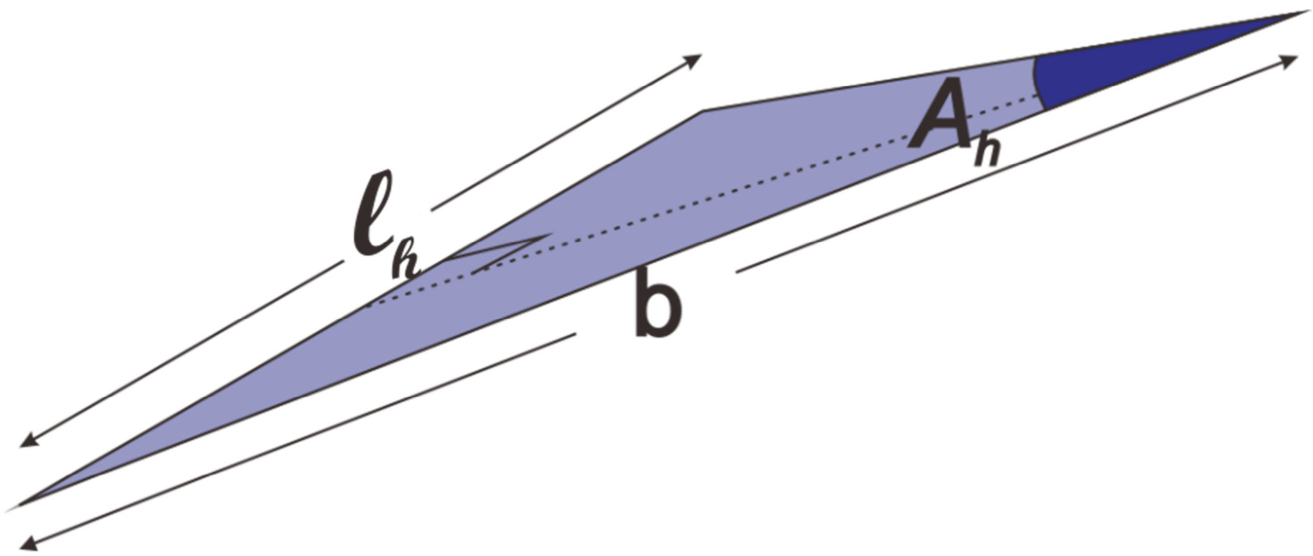

Figure 3.5 Scenario 2 - Horizontal length of view

$$\sin\frac{A_h}{2} = \frac{\frac{l_h}{2}}{b}$$

$$2b\sin\frac{A_h}{2} = l_h$$

Sub equation 3.8 into the above equation.

$$l_h = 2[\frac{x\cos\theta}{\cos(\theta - \frac{A_v}{2})}][\sin\frac{A_h}{2}] \quad (3.10)$$

Multiplying equation 3.4 and 3.5 together, we get the area.

$$Area = l_v \times l_h$$

$$= [\frac{x\cos\theta\sin A_v}{\cos(\theta - \frac{A_v}{2})\cos(\frac{A_v}{2} + \theta)}][\frac{2x\cos\theta\sin\frac{A_h}{2}}{\cos(\theta - \frac{A_v}{2})}]$$

$$= \frac{2x^2(\cos\theta)^2\sin A_v\sin\frac{A_h}{2}}{[\cos(\theta - \frac{A_v}{2})]^2\cos(\frac{A_v}{2} + \theta)}$$

Therefore, the equation to find the area of view when $\theta > \frac{A_v}{2}$ is $\frac{2x^2(\cos\theta)^2\sin A_v\sin\frac{A_h}{2}}{[\cos(\theta - \frac{A_v}{2})]^2\cos(\frac{A_v}{2} + \theta)}$.

We tried the formulas out in Microsoft Excel and yielded the following results.

x	A_h	A_v	Angle of tilt (radian)	l_v	l_h	Area of View	Angle of tilt (degrees)
50	0.698132	0.523599	0.017453293	26.799587	35.243694	944.51643	1
50	0.698132	0.523599	0.034906585	26.8136	35.080285	940.62871	2
50	0.698132	0.523599	0.506145483	31.327162	30.829524	965.80152	29
50	0.698132	0.523599	0.523598776	31.69873	30.664687	972.03163	30
50	0.698132	0.523599	0.541052068	32.091698	30.498301	978.74225	31
50	0.698132	0.523599	0.785398163	40.824829	27.925828	1140.0671	45
50	0.698132	0.523599	0.872664626	46.418887	26.838279	1245.803	50
50	0.698132	0.523599	0.959931089	54.730027	25.608788	1401.5696	55
50	0.698132	0.523599	0.977384381	56.895757	25.341567	1441.8277	56
50	0.698132	0.523599	0.994837674	59.29157	25.066113	1486.2092	57
50	0.698132	0.523599	1.012290966	61.956505	24.781862	1535.3975	58
50	0.698132	0.523599	1.029744259	64.939237	24.488204	1590.2453	59
50	0.698132	0.523599	1.047197551	68.30127	24.184476	1651.8304	60
50	0.698132	0.523599	1.064650844	72.121506	23.869957	1721.5373	61
50	0.698132	0.523599	1.082104136	76.502907	23.543859	1801.1736	62
50	0.698132	0.523599	1.099557429	81.582492	23.20532	1893.1478	63
50	0.698132	0.523599	1.117010721	87.546786	22.853395	2000.7413	64
50	0.698132	0.523599	1.134464014	94.656522	22.487048	2128.5457	65
50	0.698132	0.523599	1.221730476	171.04264	20.394453	3488.3211	70
50	0.698132	0.523599	1.396263402	-117.8599	14.053149	1656.3024	80
50	0.698132	0.523599	1.570796327	-6.03E-13	2.135E-13	-1.287E-25	90
50	0.698132	1.570796	0	100	36.397023	3639.7023	0
50	0.698132	1.570796	0.017453293	100.04571	47.539154	4756.0886	1
50	0.698132	1.570796	0.034906585	100.18312	46.736865	4682.2452	2
50	0.698132	1.570796	0.506145483	165.04773	31.119263	5136.1637	29
50	0.698132	1.570796	0.523598776	173.20508	30.664687	5311.2796	30
50	0.698132	1.570796	0.541052068	182.5813	30.214343	5516.5742	31
50	0.698132	1.570796	0.785398163	2.188E+16	24.184476	5.293E+17	45
50	0.698132	1.570796	0.802851456	-1990.454	23.762335	-47297.83	46
50	0.698132	1.570796	0.872664626	-370.1666	22.068609	-8169.063	50
50	0.698132	1.570796	0.959931089	-167.7025	19.920101	-3340.651	55
50	0.698132	1.570796	0.977384381	-149.2747	19.48349	-2908.392	56
50	0.698132	1.570796	0.994837674	-133.9046	19.043907	-2550.067	57
50	0.698132	1.570796	1.012290966	-120.8837	18.60105	-2248.564	58
50	0.698132	1.570796	1.029744259	-109.7059	18.154609	-1991.668	59
50	0.698132	1.570796	1.047197551	-100	17.704265	-1770.427	60

Figure 3.6 Table of Results

Chapter 4 - Results and Analysis

4.1 Analysis of Results

Through the use of trigonometry and equating two area formulas, we have found the area of view in terms of x, A_v, andA_h, which are all known factors.

The equation to find the area of view when $\theta = 0°$ is $4x^2(\tan\frac{A_v}{2})(\tan\frac{A_h}{2})$.

The equation to find the area of view when $\theta < \frac{A_v}{2}$ is $\frac{2x^2(\cos\theta)^2 \sin A_v \sin\frac{A_h}{2}}{[\cos(\frac{A_v}{2}-\theta)]^2\cos(\frac{A_v}{2}+\theta)}$.

The equation to find the area of view when $\theta > \frac{A_v}{2}$ is $\frac{2x^2(\cos\theta)^2 \sin A_v \sin\frac{A_h}{2}}{[\cos(\theta-\frac{A_v}{2})]^2\cos(\frac{A_v}{2}+\theta)}$.

When $\theta = \frac{A_v}{2}$, the formulas for both scenarios 1 and 2 can be used.

Furthermore, $[\cos(\theta-\frac{A_v}{2})]^2 = [\cos(\frac{A_v}{2}-\theta)]^2$, so the two formulas are actually interchangeable.

However, when we tested the formulas in Microsoft Excel, we noticed that at some point, differing depending on θ and A_v, the l_v value would eventually become negative. This is likely to be because the area of view that we calculated is against a flat surface, but when the camera or phone is tilted far enough, the lowest point that the device can capture is no longer on that flat surface anymore. The larger A_v is, the smaller θ needs to be for l_v to become negative.

4.2 Limitations and Recommendations

Using these formulas, $\theta + \frac{A_v}{2}$ cannot $= 90^\circ$, because $\cos 90^\circ = 0$, which would then cause the value of the equation would be 0 and the result for the area of view to appear as such.

We would suggest finding the relationship between A_h and the area of view, as well a way around the incapability for the formula to handle situations where $\theta + \frac{A_v}{2} = 90^\circ$.

Chapter 5 - Conclusion

Within the past century, the act of taking selfies – a self-portrait picture, has been becoming increasingly popular. Selfie taking involves holding a phone or a digital camera right in front of yourself while using a hand or selfie stick for support, and it is an easy skill that most people can learn. On the contrary, taking a group selfie, also known as a 'groupie', can be a little more challenging.

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Firstly, we used trigonometric ratios to find the area of the field of view in terms of x, the vertical distance of the object from the camera lens, in an ideal situation. As the ideal situation is unlikely to happen in reality, we used the 2 most common scenarios, namely when $\theta < \frac{A_v}{2}$, and when $\theta > \frac{A_v}{2}$, where θ is the angle of tilt of the arm from the horizontal and A_v being the vertical length of the field of view. We then managed to derive an equation that works for both of the common scenarios, but not for the ideal situation. As we were only looking at the angle of tilt of the hand, we focused on the vertical angle of view.

To further this project, we recommend finding the relationship between the area of the field of view and the horizontal angle of view.

References

1. Rhett Allain (2015) “Measuring the field of view for the iPhone 6 camera” http://www.wired.com/2015/05/measuring-field-view-iphone-6-camera/
2. Nikon Corporation (2014) “DSLR Camera basics: Focal length and Picture Angle” http://imaging.nikon.com/lineup/dslr/basics/19/01.htm
3. Stapel, Elizabeth “Trigonometric Identities”, Purplemath. http://www.purplemath.com/modules/idents.htm
4. Martin Pot (2010) “Understanding your Camera: Focal Length, Field of View and Angle of View Defined” http://martybugs.net/blog/blog.cgi/learning/Field-Of-View-And-More.html
5. Paul Bourke (2003) “Field of view and focal length” http://paulbourke.net/miscellaneous/lens/