I'm not small

Submitted by S3 PLMGS (S) Students

Cheyenne Jan Lee

Liang Jing

Nicolette Lau Jia Wen

Sim Lee Ting

Paya Lebar Methodist Girls' School (Secondary)

A project presented to the Singapore Mathematical Project Festival

2016

Abstract

In mathematics, Emmert's law states that objects that generate retinal images of the same size will look different in physical size if they appear to be locating at different distance. Ames Room is one of the applications of Emmert's law where the projected height of the person changes when the location of the person changes. Ames room used this law, thus creating an optical illusion of two people of the same height are abled to be viewed of different heights and we wanted to explore into it more.

In the project carried out last year, we came up with the idea of finding the relationship of the projected height and actual height of the person.

We used trigonometry to find the relationship between the projected height of an average person and the real height of the person, ratio of the sidewalls, the angle between the back wall and the sidewall and the width between the sidewalls.

We also explored into different types of case scenarios of whenever the ratio of the walls changes to make the general equation more explicit. Our general equation may help those who want to make use of the Ames room effect especially if they want to build it such a way that the back person may achieve the target projected height.

Most Ames rooms was limited to sidewalls have a ratio of 1 : 2. Thus, we explored into that aspect even further and discovered a new general equation regardless of the ratio of the side walls which is $h_{projected} = \sqrt{\dfrac{4h_{actual}^2\left[m^2W^2+(n-m)^2y^2\tan^2\beta\right]}{W^2\left[4(n-m)^2+(n-m)^2\tan^2\beta\right]}}$.

Acknowledgement

The students involved in this report and project thank the school for this opportunity to conduct a research on the Ames Room.

They would like to express their gratitude to the project supervisor, Ms Kok Lai Fong for her guidance in the course of preparing this project.

Table of Contents

CHAPTER 3: Methodology

CHAPTER 4: Conclusion

Chapter 1 Introduction

1.1 Objective

Our objective is to find out the relationship between the projected height of a person and the actual height of the person, ratio of the parallel walls, angle between the back wall and sidewall as well as the width of the wall, in an Ames room.

1.2 Insight

We wanted to find out the projected height of an average person in a room with different measurements so museums could build Ames Rooms based on the projected height of the back man they wish to achieve.

1.3 Background

Adelbert Ames Jr., who was an American scientist, inspired the Ames room. He contributed a lot to physiology, ophthalmology, physics, psychology and philosophy. He served as a research professor at Dartmouth College, then as a director of research at the Dartmouth Eye Institute. He researched into certain aspects of binocular vision, like aniseikonia and cyclophoria. He is famous for building optical illusions of visual perception just like the Ames room.

Chapter 2: Literature Review

2.1 Overview

We found out there are certain information we would need to obtain before we can find the relationship; Emmert's law and construction of common Ames rooms. Different art museums had different sizes and measurements of Ames Room. We wanted to find out the projected height of an average person in a room with different measurements so museums could build Ames Rooms based on the projected height of the back man they wish to achieve.

2.2 Ames Room

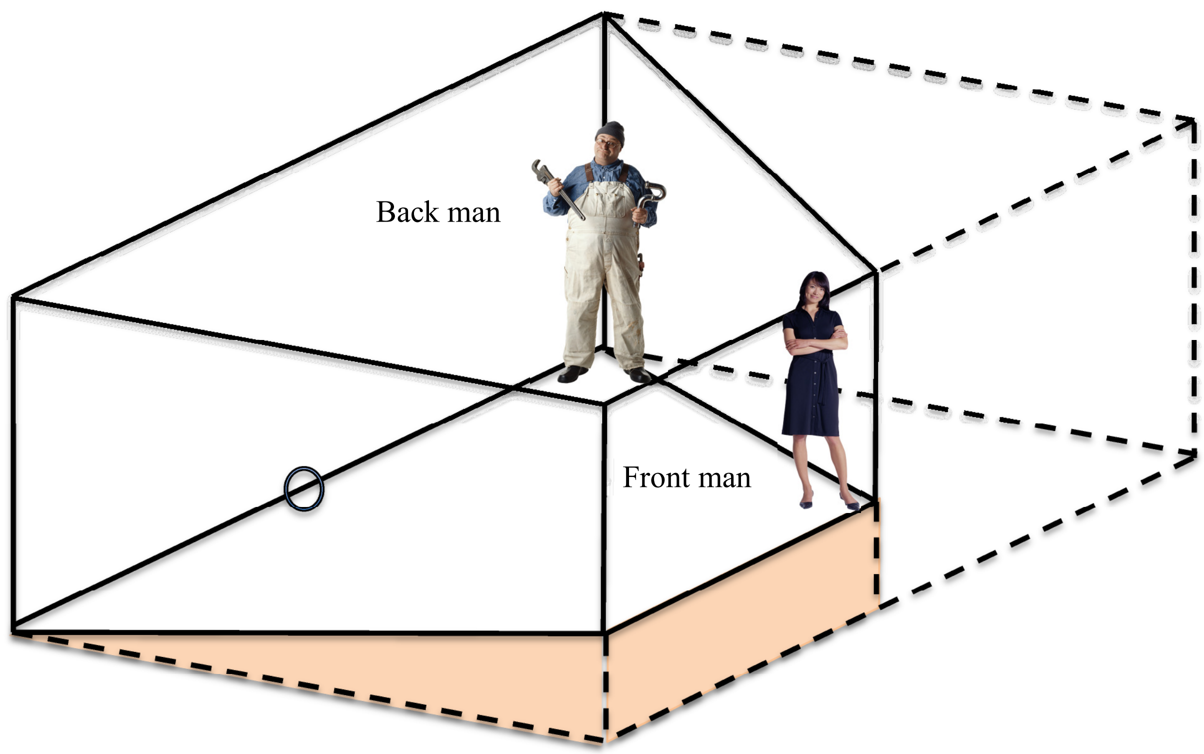

figure 2.2

The Ames room is a trapezoidal room which a person views it from a peephole and two people stand at two different corners of the room. The position of the back wall results in a giant and a dwarf image of the actual height of the back man in the Ames room.

2.2.1 Front Man

There is no projected height for the front man. When the eye visualizes the room as a rectangular room, the front man will still be seen standing at its original position. Thus the only height that can be seen is the actual height of the person, which remains constant throughout. Hence, we can conclude that the size of the front man does not change.

2.2.2 Peephole

There is always a perspective drawing on the back wall, which has only one viewing point in order to view it so the picture would not seem distorted. That only point is also the point, which makes the back wall seem perpendicular to the sidewalls. Thus, people who view this room from that point would see the room as in a rectangular shape. This viewing point is commonly found to be in the middle thus the peephole of the room is also commonly placed in the middle of the wall.

However, in rare cases where the peephole is nearer to the longer side wall, the projected height of the back man would be bigger and vice versa. This is due to the line of sight. The shorter the line of sight, the bigger the projected height of the back man and vice versa.

2.2.3 Back Man

The back man has a projected height. He stands further away from the peephole and the eye would visualize the back man to seem smaller. There are other factors, which affect the projected height of the man.

2.3 Size constancy

Size constancy is the tendency of objects to stay the same in their apparent size even though they move closer or further. The size on the retina changes as the distance from the object to the observer changes. The greater the distance the smaller the image is seen.

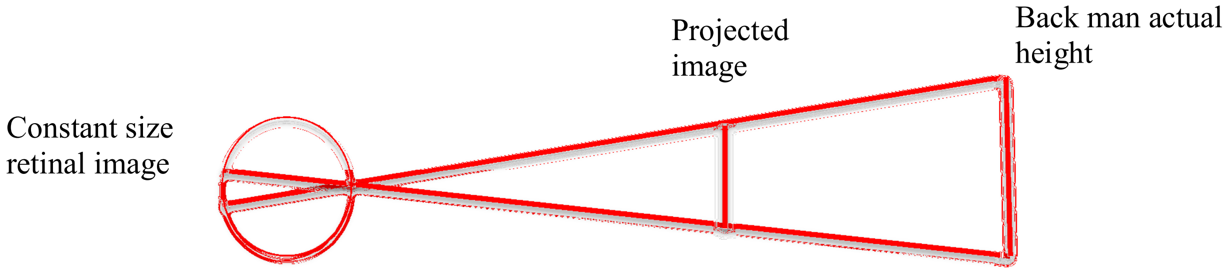

figure 2.3.1

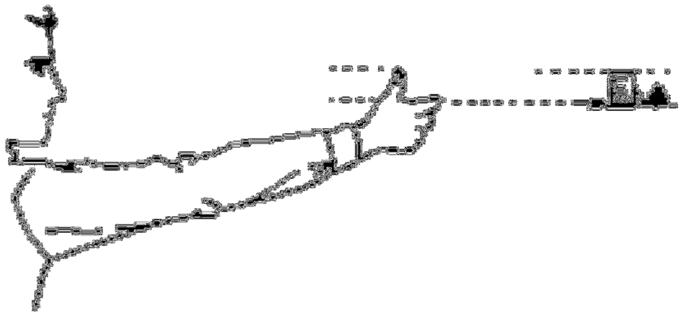

figure 2.3.2

As shown in *figure 2.3.2* above, the thumb and the building are perceived to be of the same size due to the distance away from the observer although they are obviously different in size. Since the building is located further away from the point of view, it would seem smaller, same goes for the thumb; since it is closer to the viewing point, it would seem bigger. When both objects are compared to from the viewing point, the building would seem to be the same size as the thumb.

2.3.1 Emmert's Law

This law talks about size constancy. It states that the object of the same size would look different in their linear sizes if they appear to be located at different distances. The distance determines the apparent size. The greater the distance the smaller the image is seen and vice versa.

2.3.2 How it relates to Ames Room

In the Ames Room, our eyes will see that the two people are standing in normal room, since we do not know the actual distance, we cannot determine their actual sizes.

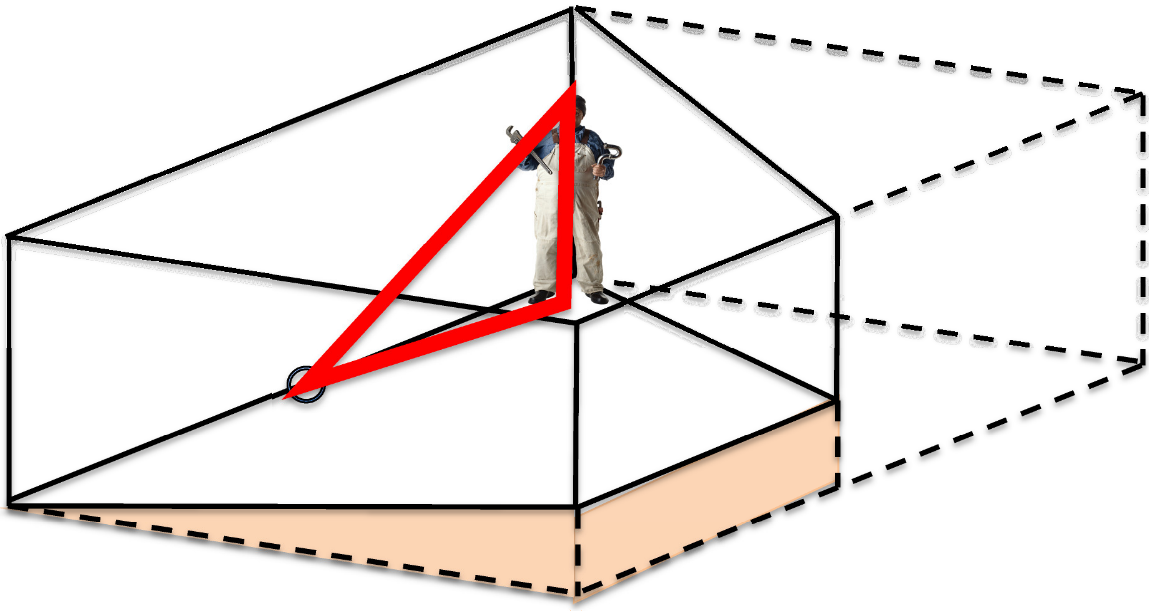

figure 2.3.2(a)

The red triangle is the visual field; it shows where the eye can see from the peephole. The projected image of the man seen from the eye is along in this triangle; this triangle is similar to the triangle in figure 2.3.1.

Chapter 3: Methodology

3.1 Overview

We would be exploring on the angle of visual field, length ratio of the sidewalls and the ratio of projected height versus actual height.

3.2 Finding the angle of visual field

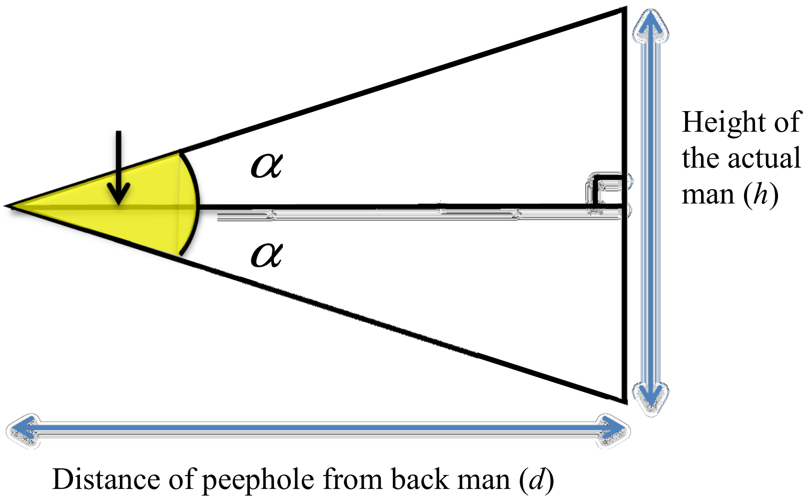

figure 3.2

Figure 3.2 depicts the red triangle in *figure 2.3.2*. We can use trigonometric functions to solve for the angle of visual field.

$$\tan\alpha = \frac{opposite}{adjacent}$$

$$\tan\alpha = \frac{\frac{1}{2}h}{d}$$

$$\tan\alpha = \frac{h}{2d}$$

$$\alpha = \tan^{-1}\frac{h}{2d}$$

$$2\alpha = 2\tan^{-1}\frac{h}{2d}$$

$$\text{Angle of visual field } = 2\tan^{-1}\frac{h}{2d}$$

Thus we can conclude that the angle of visual field of the back man is $2\tan^{-1}\frac{h}{2d}$

3.2.1 Using Length A to find the angle of visual field

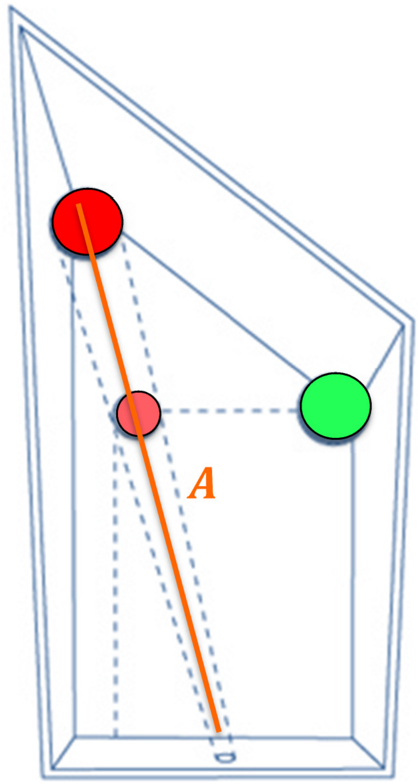

figure 3.2.1(a)

Length A is the distance from the actual position to the peephole. We can use Length A to calculate the angle of visual field of the back man.

Before calculating Length A, we take it as the ratio of the length of the left wall is to the length of the right wall is 1:2 as recommended in our readings.

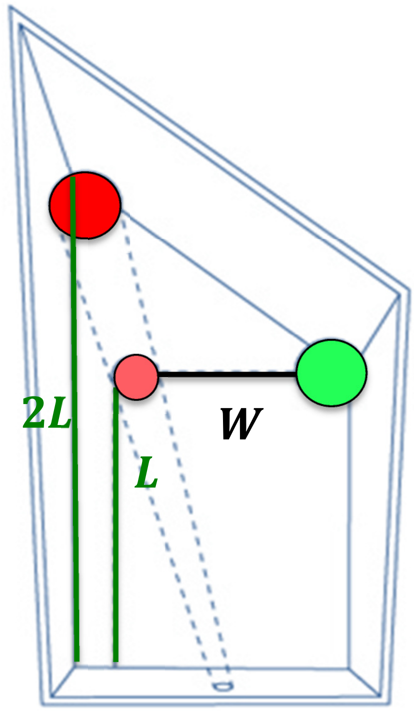

figure 3.2.1(b)

We would name the length of the left wall L and the right wall $2L$ since the ratio of their lengths is 1:2. Also, we would name the perpendicular distance between both walls to be W.

Using tangent, we can find the length L by using the angle β.

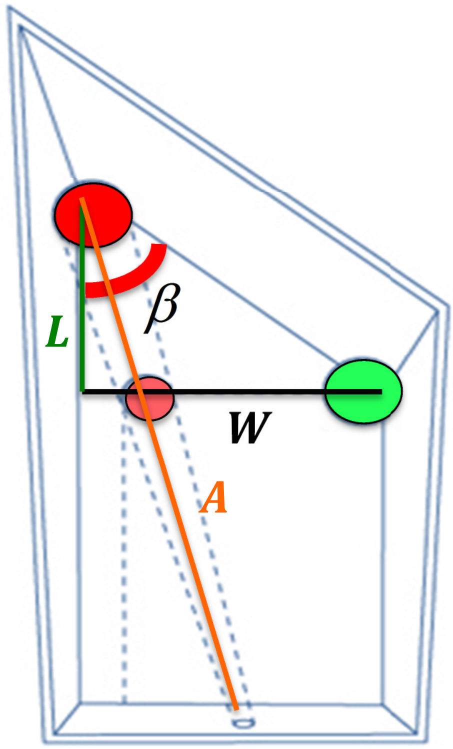

figure 3.2.1(c)

From figure 3.2.1(c), we can conclude,

$$\tan\beta = \frac{W}{L}$$

$$L = \frac{W}{\tan\beta}$$

By using Pythagoras theorem, we can find length A, which is the distance from the actual position to the peephole. Since the peephole is in the middle, we can conclude,

$$A = \sqrt{(2L)^2 + \left(\frac{W}{2}\right)^2}$$

Since we have found length A, we can use subject of the formula to find the angle of visual field

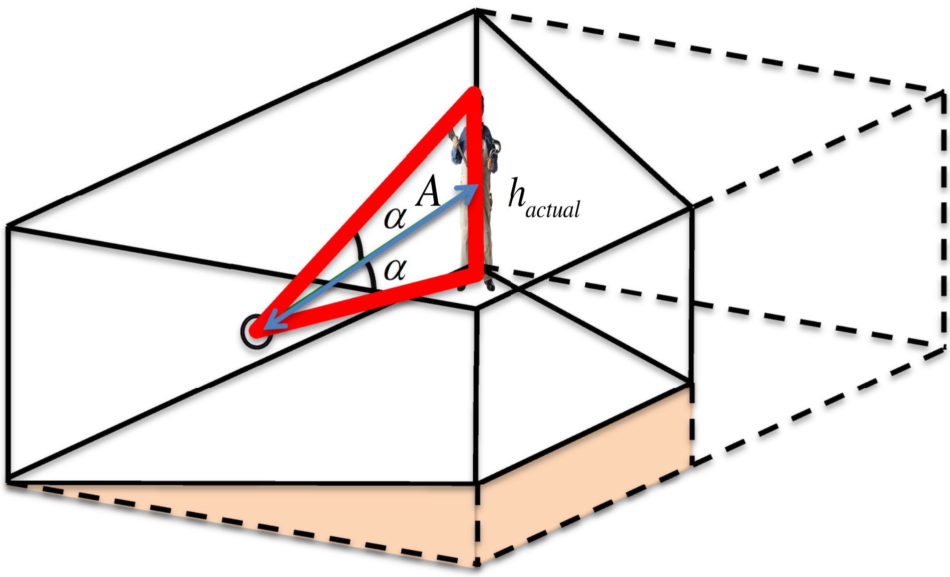

figure 3.2.1(d)

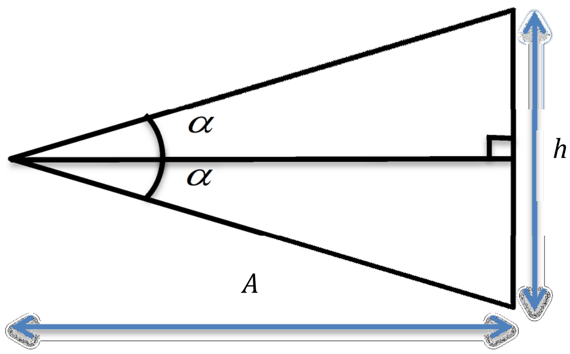

figure 3.2.1(e)

Since , $2\alpha = 2\tan^{-1}\left(\frac{h}{2d}\right)$

Therefore, $2\alpha = 2\tan^{-1}\left(\frac{h_{actual}}{2A}\right)$

Hence, Angle of visual field = $2\alpha = 2\tan^{-1}\left(\frac{h_{actual}}{2A}\right)$

3.2.2 Using Length B to find angle of visual field

We are going to use the same method to find the angle of visual field but by using length B this time.

We named the width distance between the projected image and the peephole y as it is an unknown.

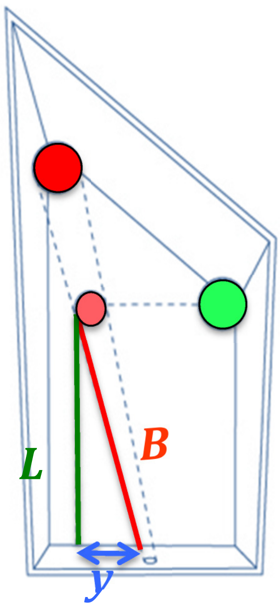

figure 3.2.2(a)

Since we know $L = \dfrac{W}{\tan\beta}$,

By using Pythagoras theorem, we can find length B,

$$B = \sqrt{L^2 + y^2}$$

Since we have found length B, we can use simultaneous equations and subject of formula to find the angle of the visual field.

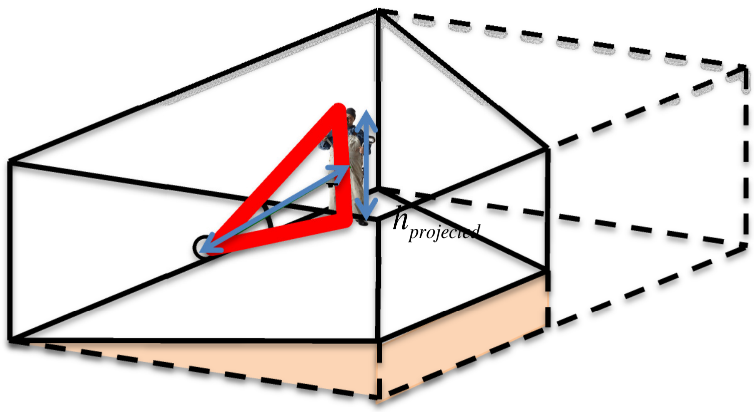

figure 3.2.2(b)

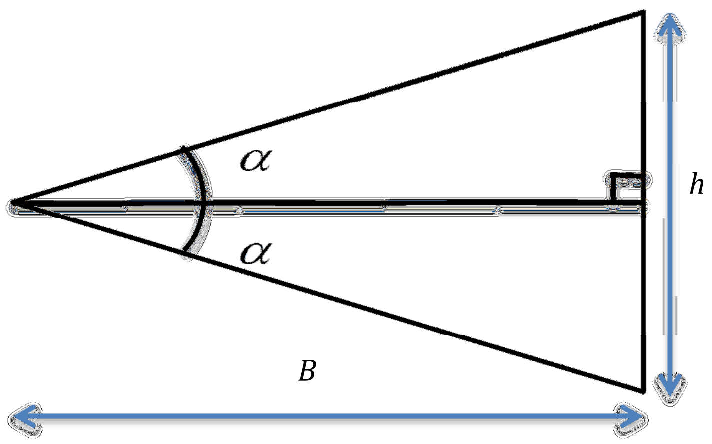

figure 3.2.2(c)

Since, $2\alpha = 2\tan^{-1}\left(\dfrac{h}{2d}\right)$

Therefore, $2\alpha = 2\tan^{-1}\left(\dfrac{h_{projected}}{2B}\right)$

Thus we can conclude, Angle of visual field = $2\alpha = 2\tan^{-1}\left(\dfrac{h_{projected}}{2B}\right)$

3.3 Relationship between projected height and actual height

When using length A,

$$\text{Angle of visual field} = 2\alpha = 2\tan^{-1}\left(\frac{h_{actual}}{2A}\right)$$

When using length B,

$$\text{Angle of visual field} = 2\alpha = 2\tan^{-1}\left(\frac{h_{projected}}{2B}\right)$$

Since they are equal, we can equate them together,

$$2\tan^{-1}\left(\frac{h_{projected}}{2B}\right) = 2\tan^{-1}\left(\frac{h_{actual}}{2A}\right)$$

$$\left(\frac{h_{projected}}{2B}\right) = \left(\frac{h_{actual}}{2A}\right)$$

$$\frac{h_{actual}}{A} = \frac{h_{projected}}{B}$$

$$h_{projected} = \frac{B \times h_{actual}}{A}$$

Now, we substitute in lengths A and B into the previous equation.

Sub in $A = \sqrt{(2L)^2 + \left(\frac{W}{2}\right)^2}$ and $B = \sqrt{L^2 + y^2}$ into $h_{projected} = \frac{B \times h_{actual}}{A}$,

Thus we can rewrite it as $h_{projected} = \frac{h_{actual}\sqrt{L^2 + y^2}}{\sqrt{(2L)^2 + \left(\frac{W}{2}\right)^2}}$

Then, we substitute L into the new equation.

Since $L = \dfrac{W}{\tan\beta}$,

Thus,

$$h_{projected} = \frac{h_{actual}\sqrt{L^2 + y^2}}{\sqrt{(2L)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{W}{\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{2W}{\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{W}{\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{2\times W}{\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h^2_{projected} = \frac{h^2_{actual}\left(\frac{W^2 + y^2\tan^2\beta}{\tan^2\beta}\right)}{\frac{4W^2}{\tan^2\beta} + \frac{W^2}{4}}$$

$$h^2_{projected} = \frac{h^2_{actual}\left(\frac{W^2 + y^2\tan^2\beta}{\tan^2\beta}\right)}{\frac{4W^2}{\tan^2\beta} + \frac{W^2}{4}}$$

$$h^2_{projected} = \frac{h^2_{actual}(W^2 + y^2\tan^2\beta)}{\frac{16W^2 + W^2\tan^2\beta}{4}}$$

$$h^2_{projected} = \frac{4h^2_{actual}(W^2 + y^2\tan^2\beta)}{W^2(16 + \tan^2\beta)}$$

Thus we can get the projected height just by square root.

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(W^2 + y^2 \tan^2 \beta)}{W^2(16 + \tan^2 \beta)}}$$

3.4 Ratio of the sidewalls

However, this equation can only be used with the condition of the ratio of the length of the sidewalls is 1:2. Let *n* meters the length of the longer sidewall and *m* meters be the length of the shorter wall,

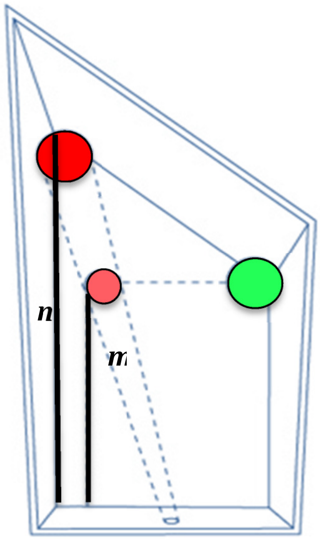

figure 3.4

The angle of visual field would remain constant while only the lengths *A* and *B* may vary based on the ratio sidewalls.

3.4.1 when the ratio of the sidewalls is 1: *n*

We are trying to find a pattern of the final equations from 3 different case scenarios when the ratio of the sidewalls is different. In case scenario 1, we would use the ratio 1:3, thus Length L would be $L = \dfrac{W}{2\tan\beta}$

By Pythagoras theorem,

$$A = \sqrt{(3L)^2 + (\frac{W}{2})^2} \qquad \text{and} \qquad B = \sqrt{L^2 + y^2}$$

From the relationship between A, B, projected height and actual height before, we substitute A and B into it.

$$h_{projected} = \frac{B \times h_{actual}}{A}$$

$$h_{projected} = \frac{h_{actual}\sqrt{L^2 + y^2}}{\sqrt{(3L)^2 + \left(\frac{W}{2}\right)^2}}$$

Followed by substituting L in,

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{W}{2\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{3W}{2\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \sqrt{\frac{h_{actual}^2(W^2 + 4y^2\tan^2\beta)}{\frac{36W^2 + 4W^2\tan^2\beta}{4}}}$$

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(W^2 + 4y^2\tan^2\beta)}{W^2(36 + 4\tan^2\beta)}}$$

In case scenario 2, we will use the ratio of 1:4, thus Length L would be $L = \dfrac{W}{3\tan\beta}$

By Pythagoras theorem,

$$A = \sqrt{(4L)^2 + (\frac{W}{2})^2} \qquad \text{and} \qquad B = \sqrt{L^2 + y^2}$$

Substitute A and B in to solve for the projected height,

$$h_{projected} = \frac{h_{actual}\sqrt{L^2 + y^2}}{\sqrt{(4L)^2 + \left(\frac{W}{2}\right)^2}}$$

Followed by substituting the L we just found,

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{W}{3\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{4W}{3\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \sqrt{\frac{h_{actual}^2(W^2 + 9y^2\tan^2\beta)}{\frac{64W^2 + 9W^2\tan^2\beta}{4}}}$$

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(W^2 + 9y^2\tan^2\beta)}{W^2(64 + 9\tan^2\beta)}}$$

In the first scenario, we managed to get

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(W^2 + 4y^2\tan^2\beta)}{W^2(36 + 4\tan^2\beta)}}$$

In the second scenario, we achieved this

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(W^2 + 9y^2\tan^2\beta)}{W^2(64 + 9\tan^2\beta)}}$$

From both equations we managed to pick up a pattern in them which is

$$h_{projected} = \sqrt{\frac{4h_{actual}^2\left[W^2 + (n-1)^2 y^2 \tan^2\beta\right]}{W^2\left[4n^2 + (n-1)^2 \tan^2\beta\right]}}$$

3.4.2 When the ratio of the sidewalls is 2:n

We would use the exact same steps to find the general equation for when the ratio of the sidewalls is 2:n. We would also use 2 case scenarios using different ratios but the ratios still fulfil the requirement of 2 : *n*.

In case scenario 1, we would use the ratio 2:3, $L = \frac{W}{\tan\beta}$

By Pythagoras theorem, we can find the lengths of *A* and *B*,

$$A = \sqrt{(3L)^2 + (\frac{W}{2})^2} \qquad \text{and} \qquad B = \sqrt{(2L)^2 + y^2}\,.$$

By substitution,

$$h_{projected} = \frac{h_{actual}\sqrt{(2L)^2 + y^2}}{\sqrt{(3L)^2 + \left(\frac{W}{2}\right)^2}}$$

Since $L = \frac{W}{\tan\beta}$,

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{2W}{\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{3W}{\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \sqrt{\frac{h_{actual}^2 (4W^2 + y^2 \tan^2 \beta)}{\frac{9W^2 + W^2 \tan^2 \beta}{4}}}$$

$$h_{projected} = \sqrt{\frac{4h_{actual}^2 (4W^2 + y^2 \tan^2 \beta)}{W^2 (9 + \tan^2 \beta)}}$$

In the second scenario, we will use the ratio 2:5, thus $L = \frac{W}{3\tan\beta}$

By Pythagoras theorem,

$$A = \sqrt{(5L)^2 + (\frac{W}{2})^2} \qquad \text{and} \qquad B = \sqrt{(2L)^2 + y^2}$$

Substitute both A and B into $h_{projected} = \frac{B \times h_{actual}}{A}$

$$h_{projected} = \frac{h_{actual}\sqrt{(2L)^2 + y^2}}{\sqrt{(5L)^2 + \left(\frac{W}{2}\right)^2}}$$

Substitute L,

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{2W}{3\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{5W}{3\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \sqrt{\frac{h_{actual}^2 (4W^2 + 9y^2 \tan^2 \beta)}{\frac{100W^2 + 9W^2 \tan^2 \beta}{4}}}$$

$$h_{projected} = \sqrt{\frac{4h_{actual}^2 (4W^2 + 9y^2 \tan^2 \beta)}{W^2 (100 + 9\tan^2 \beta)}}$$

From scenario 1, we get,

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(4W^2 + y^2\tan^2\beta)}{W^2(9+\tan^2\beta)}}$$

From the second scenario, we get,

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(4W^2 + 9y^2\tan^2\beta)}{W^2(100+9\tan^2\beta)}}$$

From both final equations, we managed to find a pattern and concluded with this general equation for when the sidewalls are in the ratio of 2 : *n*.

$$h_{projected} = \sqrt{\frac{4h_{actual}^2\left[(2^2)W^2 + (n-2)^2 y^2\tan^2\beta\right]}{W^2\left[4(n-2)+(n-2)^2\tan^2\beta\right]}}$$

3.4.3 When the ratio of the side walls is 3:n

We shall repeat the exact same steps again for two different scenarios when using ratios of the sidewalls, which satisfy the 3:n condition.

In scenario 1, we will use the ratio 3 : 4, thus L would be $L = \dfrac{W}{\tan\beta}$

By Pythagoras theorem,

$$A = \sqrt{(4L)^2 + (\frac{W}{2})^2} \qquad \text{and} \qquad B = \sqrt{(3L)^2 + y^2}$$

Again, we substitute *A* and *B* into the previous equation we derived at,

$$h_{projected} = \frac{h_{actual}\sqrt{(3L)^2 + y^2}}{\sqrt{(4L)^2 + \left(\frac{W}{2}\right)^2}}$$

Then substitute L,

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{3W}{\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{4W}{\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \sqrt{\frac{h_{actual}^2(9W^2 + y^2\tan^2\beta)}{\frac{64W^2 + W^2\tan^2\beta}{4}}}$$

$$h_{projected} = \sqrt{\frac{4h_{actual}^2(9W^2 + y^2\tan^2\beta)}{W^2(64 + \tan^2\beta)}}$$

In the second case scenario, we will use the ratio of 3:5, thus $L = \frac{W}{2\tan\beta}$

By Pythagoras theorem,

$$A = \sqrt{(5L)^2 + (\frac{W}{2})^2} \qquad \text{and} \qquad B = \sqrt{(3L)^2 + y^2}$$

Substitute both into the same equation, which previously used,

$$h_{projected} = \frac{h_{actual}\sqrt{(3L)^2 + y^2}}{\sqrt{(5L)^2 + \left(\frac{W}{2}\right)^2}}$$

Then substitute *L*,

$$h_{projected} = \frac{h_{actual}\sqrt{\left(\frac{3W}{2\tan\beta}\right)^2 + y^2}}{\sqrt{\left(\frac{5W}{2\tan\beta}\right)^2 + \left(\frac{W}{2}\right)^2}}$$

$$h_{projected} = \sqrt{\frac{h_{actual}^2(9W^2 + 4y^2\tan^2\beta)}{\frac{25W^2 + 4W^2\tan^2\beta}{4}}}$$

$$h_{projected}=\sqrt{\frac{4h_{actual}^{2}(9W^{2}+4y^{2}\tan^{2}\beta)}{W^{2}(25+4\tan^{2}\beta)}}$$

From the first scenario, we got,

$$h_{projected}=\sqrt{\frac{4h_{actual}^{2}(9W^{2}+y^{2}\tan^{2}\beta)}{W^{2}(64+\tan^{2}\beta)}}$$

From the second scenario, we got

$$h_{projected}=\sqrt{\frac{4h_{actual}^{2}(9W^{2}+4y^{2}\tan^{2}\beta)}{W^{2}(25+4\tan^{2}\beta)}}$$

From both scenarios, we can conclude there is a pattern and have derived the general equation of when the ratio of the sidewalls fulfill the condition of them being in 3:n.

$$h_{projected}=\sqrt{\frac{4h_{actual}^{2}\left[(3^{2})W^{2}+(n-3)^{2}y^{2}\tan^{2}\beta\right]}{W^{2}\left[4(n-3)+(n-3)^{2}\tan^{2}\beta\right]}}$$

3.4.4 General Equation

We noticed that there was a pattern among the general equations we derived at. The following table shows the general equations we got from the different ratios of the sidewalls.

Ratio of the sidewalls	General equation derived at
$1 : n$	$h_{projected} = \sqrt{\dfrac{4h_{actual}^2\left[W^2 + (n-1)^2 y^2 \tan^2\beta\right]}{W^2\left[4n^2 + (n-1)^2 \tan^2\beta\right]}}$
$2 : n$	$h_{projected} = \sqrt{\dfrac{4h_{actual}^2\left[(2^2)W^2 + (n-2)^2 y^2 \tan^2\beta\right]}{W^2\left[4(n-2) + (n-2)^2 \tan^2\beta\right]}}$
$3 : n$	$h_{projected} = \sqrt{\dfrac{4h_{actual}^2\left[(3^2)W^2 + (n-3)^2 y^2 \tan^2\beta\right]}{W^2\left[4(n-3) + (n-3)^2 \tan^2\beta\right]}}$

We noticed that the equations were all similar and came up with an overall general equation with n meters as the longer side wall and m meters as the shorter side of the wall.

Hence, we conclude that

$$h_{projected} = \sqrt{\frac{4h_{actual}^2\left[m^2W^2 + (n-m)^2 y^2 \tan^2\beta\right]}{W^2\left[4(n-m)^2 + (n-m)^2 \tan^2\beta\right]}}$$

Chapter 4: Conclusion

4.1 Conclusion

We studied Emmert's Law and realized it was related to the optical illusion found in the Ames room. We noticed the effect of this optical illusion was due to the angle of the back wall and the perspective daring on it. The angle of the back wall increases the distance between the back man and the peephole thus the person would seem smaller while the other person is bigger than the other due to the shorter distance between the peephole and the front man.

We used trigonometry to find the Lengths of certain parts of the room. Then we went on to express the angle of visual field using different lengths like lengths *A* and *B*. Since the angle of visual field remains constant, we used simultaneous equations and equated them together to get the first general equation. However, it was limited only to those Ames rooms whose sidewalls have a ratio of 1:2. Thus, we explored into that aspect even further and discovered a new general equation regardless of the ratio of the side walls which is

$$h_{projected} = \sqrt{\frac{4h_{actual}^2\left[m^2W^2 + (n-m)^2 y^2 \tan^2\beta\right]}{W^2\left[4(n-m)^2 + (n-m)^2 \tan^2\beta\right]}}$$

Reference:

11 October:

1. http://psylux.psych.tu-dresden.de/i1/kaw/diverses%20Material/www.illusionworks.com/html/ames_room.html
2. http://www.instructables.com/id/Ames-Room-Optical-Illusion/
3. http://psychology.about.com/od/sensationandperception/ig/Optical-Illusions/Ames-Room-Illusion.htm
4. http://www.newworldencyclopedia.org/entry/Ames_room
5. http://www.richannel.org/tales-from-the-prep-room-the-ames-room
6. http://lite.bu.edu/inkjet-science/pdfs/ProjectLITEAmesRoom.pdf
7. http://psychsciencenotes.blogspot.sg/2010/10/runeson-ames-room-and-irrelevance-of.html
8. http://www.alleydog.com/glossary/definition.php?term=Ames%20Room
9. http://www.whiterabbitdesigncompany.com/ames-room.html

12 October:

10. http://perspectiveresources.blogspot.sg/2011/06/how-to-construct-ames-room.html
11. https://prezi.com/dcvi6rqc7x1q/the-ames-room-illusion/
12. http://www.pc.rhul.ac.uk/staff/J.Zanker/illusions/ames_room.pdf
13. http://demonstrations.wolfram.com/AmesRoom/
14. http://www.math.utah.edu/~treiberg/Perspect/Perspect.htm
15. http://www.math.nus.edu.sg/aslaksen/projects/kh2-urops.pdf
16. http://euler.slu.edu/escher/index.php/Depth_and_Perspective
17. http://www.mathaware.org/mam/2014/calendar/perspective.html

15 October:

18. https://courses.cs.washington.edu/courses/cse576/book/ch12.pdf
19. http://www.bbc.com/future/bespoke/story/20150130-how-your-eyes-trick-your-mind/index.html
20. http://researchnews.osu.edu/archive/eyefolow.htm
21. https://plus.maths.org/content/visual-curiosities-and-mathematical-paradoxes
22. http://weburbanist.com/2009/12/03/trick-of-the-eye-delightfully-deceptive-works-of- art/
23. http://numb3rs.wolfram.com

17 November:

24. http://link.springer.com/article/10.3758%2FBF03210442
25. http://www.lightspeedmagazine.com/wp-content/uploads/2014/05/ames-room-diagram.jpg
26. http://ruccs.rutgers.edu/~ikovacs/SandP2000/prepIII_1.html