RATE OF FLUID FLOW THROUGH POROUS MEDIA

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Abstract

Fluid flow through the planet's subsurface is critically important to many ecological and economic activities. Subsurface flow is an important part of the entire water cycle. In fact, the United States Geologic Survey estimates that, worldwide, there is 30 times more groundwater stored in aquifers than is found in all the fresh-water lakes and rivers. Unfortunately, contaminants sometimes leach out of storage sites, either from above ground tanks or through underground containment barriers, and travel through the Earth. Petroleum and natural gas are extracted from the subsurface by inducing these fluids to flow to production wells. To mitigate the effects of greenhouse gas accumulation in the atmosphere, technologies are being developed to sequester carbon dioxide, extracted from power plant emissions, in deep underground reservoirs. These and many other situations make it important that we have the ability to simulate the flow of fluids in the subsurface. Our essay focuses on using Darcy's Law to calculate the rate of fluid flow through a porous medium, and to estimate the rate of water flow through a large area consisting of different porous media.

1. Introduction

Fluid flow through the planet's subsurface is critically important to many ecological and economic activities. Subsurface flow is an important part of the entire water cycle. In this essay we would be exploring Darcy's Law so as to give us an insight of the rate of fluid flow through porous media.

2. Rate of water flow through porous media

Darcy's law is a phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth sciences, particularly in hydrogeology. Prediction of the movement of the fluids is crucial as this information would help to protect the natural ecosystem and human health, optimize the economic benefit of the world's underground natural fluid resources, and minimize unintended impact on the natural environment.

2.1 Darcy's law

Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance.

$$Q = \frac{-kA}{\mu}\frac{(P_b - P_a)}{L}$$

The total discharge, Q (units of volume per time, e.g., m^3/s) is equal to the product of the permeability of the medium, k (m^2), the cross-sectional area to flow, A (units of area, m^2), and the pressure drop (P_b - P_a), all divided by the viscosity, μ (Pa·s) and the length over which the pressure drop is taking place (m). The negative sign is needed because fluid flows from high pressure to low pressure. If the change in pressure is negative (where $P_a > P_b$), then the flow will be in the positive '*x*' direction.

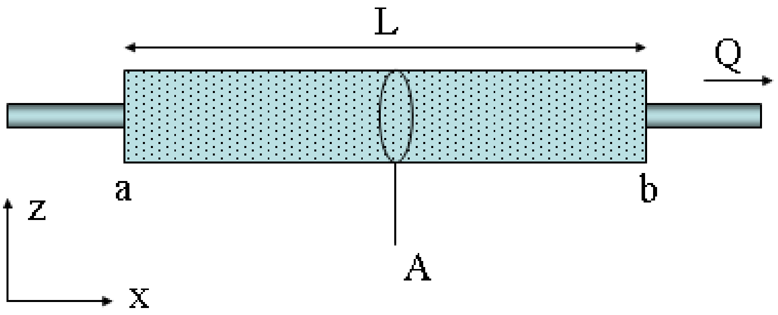


Diagram showing definitions and directions for Darcy's law.

Dividing both sides of the equation by the area and using more general notation leads to

$$q = \frac{-k}{\mu}\nabla P,$$

where q is the flux (discharge per unit area, with units of length per time, m/s) and is the pressure gradient vector (Pa/m). This value of flux, often referred to as the Darcy flux, is not the velocity which the water traveling through the pores is experiencing. The pore velocity (v) is related to the Darcy flux (q) by the porosity (n). The flux is divided by porosity to account for the fact that only a fraction of the total formation volume is available for flow. The pore velocity would be the velocity a conservative tracer would experience if carried by the fluid through the formation.

$$v = \frac{q}{n}$$

Darcy's law is a simple mathematical statement which neatly summarizes several familiar properties that groundwater flowing in aquifers exhibits, including:

- if there is no pressure gradient over a distance, no flow occurs (these are hydrostatic conditions),
- if there is a pressure gradient, flow will occur from high pressure towards low pressure

(opposite the direction of increasing gradient - hence the negative sign in Darcy's law),

- the greater the pressure gradient (through the same formation material), the greater the discharge rate, and
- the discharge rate of fluid will often be different — through different formation materials (or even through the same material, in a different direction) — even if the same pressure gradient exists in both cases.

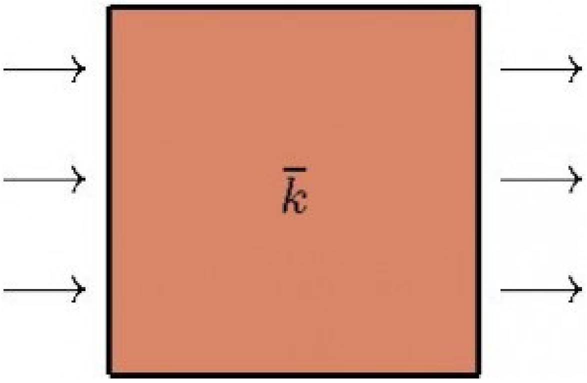
Darcy's law is only valid for slow, viscous flow; fortunately, most groundwater flow cases fall in this category. Typically any flow with a Reynolds number less than one is clearly laminar, and it would be valid to apply Darcy's law. Experimental tests have shown that flow regimes with Reynolds numbers up to 10 may still be Darcian, as in the case of groundwater flow. The Reynolds number (a dimensionless parameter) for porous media flow is typically expressed as

$$Re = \frac{\rho v d_{30}}{\mu}$$

where ρ is the density of water (units of mass per volume), v is the specific discharge (not the pore velocity — with units of length per time), d30 is a representative grain diameter for the porous media (often taken as the 30% passing size from a grain size analysis using sieves - with units of length), and μ is the viscosity of the fluid.

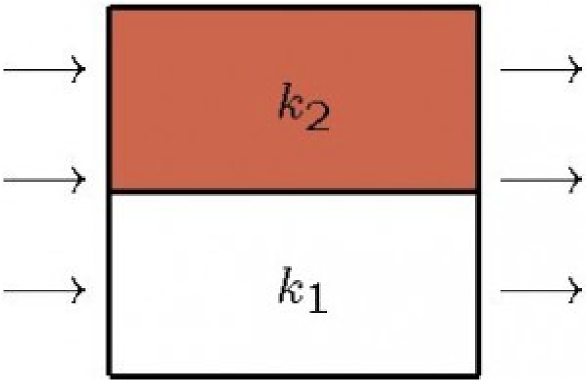
2.1. To approximate the average flow simplify the geologic structure of the porous medium

It is extremely difficult to simulate the flow of fluid through a natural porous medium like a groundwater aquifer or a petroleum reservoir because of difficulties like extreme heterogeneity of a natural rock formation. This is where mathematics comes in. Since we cannot resolve all the details of the flow, we can approximate the average flow within each grid cell, by simplifying the geologic structure of the porous medium.

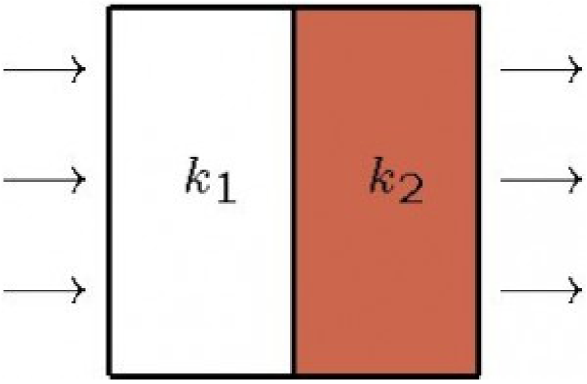


The heterogeneous porous medium is replaced by a homogeneous one with an average permeability k, chosen so that the average amount of fluid flow is the same.

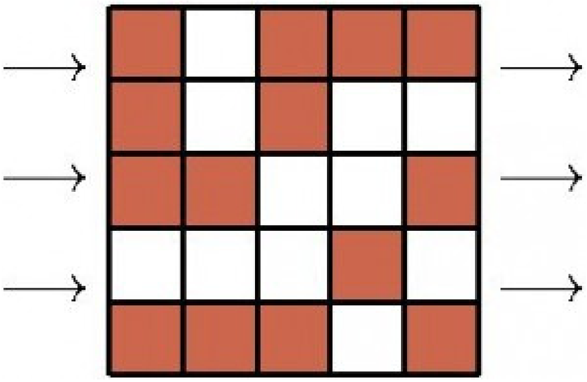
When the flow is going along with the geologic layers, the correct way to average the permeability is to take the usual arithmetic average. In this case, (k1 + k2)/2.



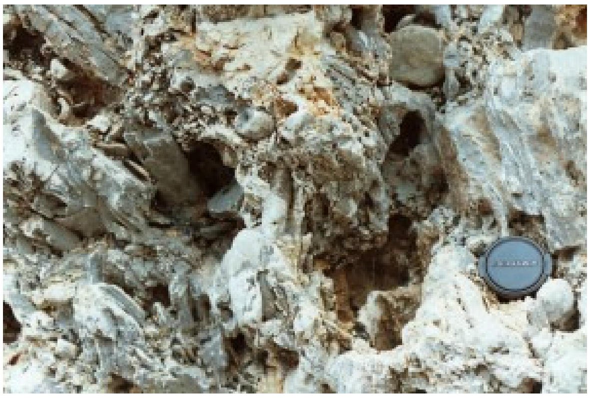
When fluid flows so that it cuts through the layers, the arithmetic average permeability does not give the correct average fluid flow. This is perhaps easy to see by considering the possibility that one of the layers becomes impermeable, say k1 = 0, so that fluid cannot flow through it. In this case, there will be no flow through the entire grid cell. But the arithmetic average is k2/2 > 0, and we should expect some fluid flow through the grid cell. The correct average to take is the harmonic average, which is the reciprocal of the arithmetic average of the reciprocals, i.e., 2/(1/k1 + 1/k2).



The harmonic average is always less than or equal to the arithmetic average. But for a genuinely heterogeneous porous medium, more sophisticated averaging techniques must be used. The mathematical theory of homogenization is an example. It can be proven that the correct average always lies between the harmonic and arithmetic averages, so our layered case actually gives the extreme cases of the problem. In fact, homogenization is also able to account for the fact that porous media may be anisotropic, i.e., they do not behave the same in every direction. The diagram below is anisotropic, we can see that as fluid tries to flow from left to right, it will also tend to flow upwards a bit as well.



These averaging techniques work well for even very complex geology. For example, consider the heterogeneous rock shown below, which is a complex mixture of fossil remains, limestone, sediments, and vugs, which are large open spaces in the rock.



3. Conclusion

Darcy's law allows mankind to approximate the average flow and simplify the geologic structure of porous medium. The ability to understand fluid flow through the planet’s subsurface would help us appreciate the earth’s ecology system and to put in measures to remove stress and strain caused by mankind to our planet earth. More complex numerical models have also been developed by mathematicians that not only give the correct average flow, but also attempt to recover some of the small scale variability of the flow.

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