EXTREME STREAMING

Submitted by S3 PLMGS (S) Students

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Abstract

Streaming is a tedious process carried out at the end of the secondary two cohorts' school year. In this process, the secondary two pupils are asked to choose subjects they would like to do for their GCE O levels. After having chosen their desired subjects, students are then streamed and placed into classes. Streaming is based firstly by merit, followed by choice. As there are many pupils involved when streaming, we aim to come up with a method that allows teachers to lighten their burden and make this process fast and accurate. The method we have come up with makes use of sets and the max-flow min-cut theorem and sets language and notation to help in the streaming of pupils.

Sets are collections of objects described with mathematics. The max-flow min-cut theorem is usually expressed as a directed graph. The concept is that for any network with a single source, the maximum feasible flow from source to sink is equal to the minimum cut value for any of the vertices of the directed graph.

Sets would be applied and the students would be arranged in different sets according to the group of sciences chosen. Each class would contain options determined by the sets. We would use the max-flow min-cut theorem to find out who will be accepted into which class. The max-flow min-cut would help us direct the students into their desired options. This is the main outline of our streaming process.

Acknowledgement

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They would like to express their gratitude to the project supervisor, Ms Kok Lai Fong for her guidance in the course of preparing this project.

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Chapter 1: Introduction

1.1 Objectives

Our objective is to find methods or equations that would aid the teachers during the secondary two pupils streaming in our school. We plan to find methods that help assign the pupils into their respective classes based on their options and academic results and their first choice.

1.2 Problem

The problem we are tackling is that streaming is a very tiring and time-consuming process. Streaming is done manually and hence, human error could occur. When human errors occur, teachers might miss out certain pupils and causing the entire streaming process to be repeated again.

1.3 Background

Streaming is the allocation of students into different categories based on merits and choice. The Singapore education system requires all secondary two pupils to choose a desired combination of subjects at the end of their academic year. This would then determine the subjects they would be doing for their GCE O level examination. This is an important process as it is the first step of specialization in certain fields of professions. Such as the requirement of taking pure physics at secondary school level to further study physics and become an engineer.

Chapter 2: Literature Review

2.1 Overview

Our literature review consists mainly of two theorems, sets language and notation, and the max-flow min-cut theorem. Set language and notation, other wise known as sets allows the least number of sets to be formed including any element present. The max–flow min-cut theorem directs the flow to a final result and reduces the cut.

2.2 Set Language And Notation

In everyday life, we speak of collection of objects such as a gaggle of geese, a pair of gloves and so on and so forth. Similarly in mathematics, the term "set" is used to describe all these.

2.2.1 Introduction

A group of objects is called a set. The individual objects in a set are called an element of a set. An element of a set can be expressed as such:

Set = {element present in the set} – (1)

Sets can be expressed as Venn diagrams as well. An example can be seen in Figure 2.2.1

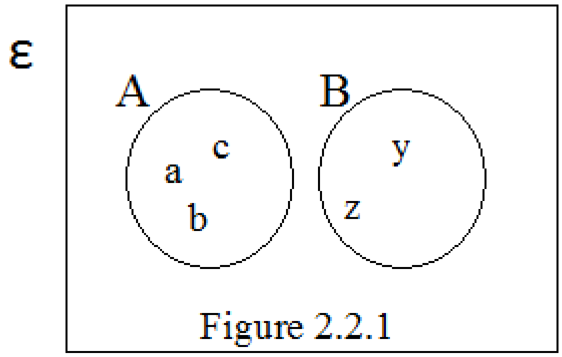


Figure 2.2.1 Example of Sets expressed as Venn Diagrams

The rectangle is the set that contains all the sets, set A and set B. The rectangle, denoted by ε, is known as the universal set. Therefore we can say that

ε = {a, b, c, z, y}

A = {a, b, c}

B = {y, z}

2.2.2 Complement of a set

Suppose ε = {integers from 1 to 5}, A = {odd numbers in the range of 1 to 5} and B = {even numbers in the range of 1 to 5}. This can be seen in Figure 2.2.2

ε = {1, 2, 3, 4, 5}

A = {1, 3, 5}

B = {2, 4}

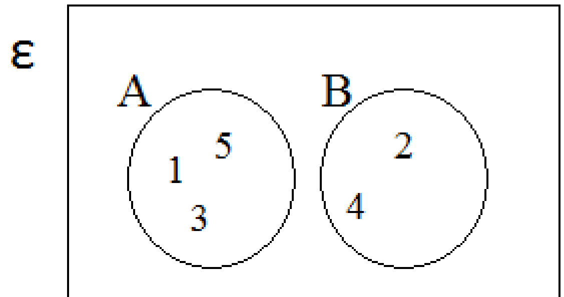


Figure 2.2.2 Example of a compliment of a set

The elements in ε are either odd or even numbers. They have no common elements and elements that are not in A, are in B. Therefore we can say that A is complement of B. Otherwise denoted by A' = B. Simply put, complement A means the elements outside of set A.

2.2.3 Subsets

Suppose ε = {integers from 1 to 7}, A = {odd numbers in the range of 1 to 7} and B = {prime numbers in the range of 1 to 7}. This can be seen in Figure 2.2.3

ε = {1, 2, 3, 4, 5, 6, 7}

A = {1, 3, 5}

B = {2}

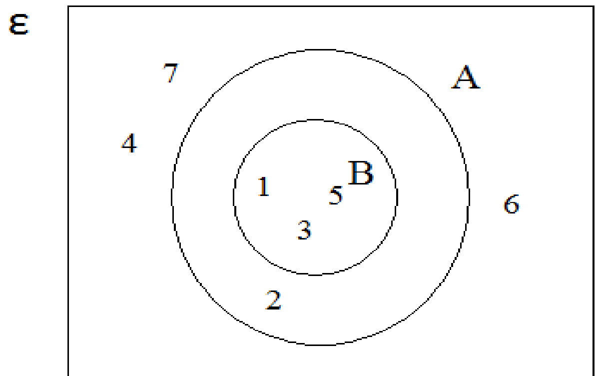


Figure 2.2.3 B as a subset of A

Each element of B is also an element of A. Therefore we can say that B is a subset of A. This can be denoted by B $\subset$A or B $\subseteq$A. $\subseteq$ Expresses the idea " includes" or "contains" and $\subset$ shows proper inclusion. Thus B $\subset$A means that B is a proper subset of A, implying that B has at least one element less that A

2.2.4 Intersection of sets

The intersection of sets A and B is the set of elements that are common to both A and B. It is denoted by A∩B. Figure 2.2.4 illustrates this.

Suppose ε = {integers from 1 to 6}, A = {1, 2, 3, 5} and B = {2, 3, 4, 6}. Then A∩B = {2, 3} as 2 and 3 belong to both set A and B.

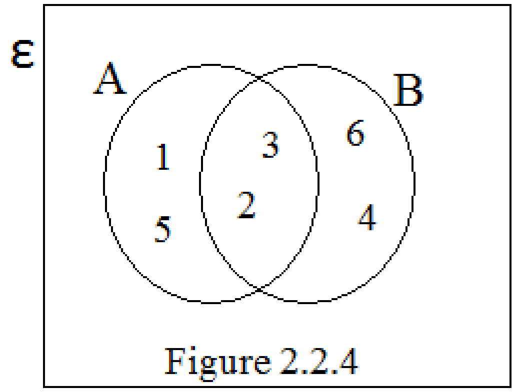


Figure 2.2.4

A∩B

The over lapping region represents A∩B

2.2.5 Union Of Sets

The union of sets A and B is the elements that are in A, or in B, or in both A and B. It is denoted by A ∪ B. This can be illustrate by Figure 2.2.5

Suppose ε = {integers from 1 to 6}, A = {1, 2, 3, 5} and B = {2, 3, 4, 6}. Then A ∪ B ={1, 2, 3, 4, 5, 6}

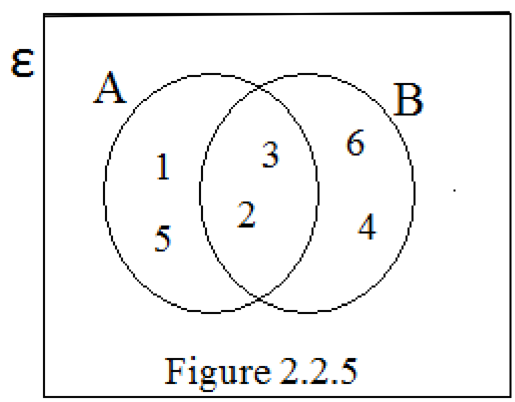


Figure 2.2.5

A ∪ B

2.3 Max-flow Min-cut Theorem

The max-flow min-cut theorem is a network, N, that is a directed graph with S being the source and T being the sink of N. A flow is a mapping of the amount that passes along the edge, the line that branches outwards from the source, S, passes through the vertex and ends at the sink, T. Below the vertex, there is another arrow that points any excess flow down to the next vertex. Figure 2.3 illustrates an example of the max-flow min-cut theorem.

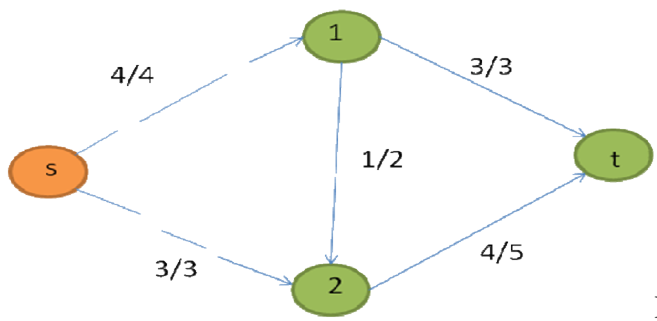


Figure 2.3

A flow is subject to two constraints, mainly, the capacity constraint and the conservation of flows. The capacity of the edge is the maximum amount of flow that is allowed to pass through the vertex and to the sink, T. The conservation of flow is the amount that would be passing through the vertex. A cut is the edge that starts from s and ends at the vertex.

The numerator before the vertex shows the amount of flow wanting to pass through the vector. The denominator on the same side shows the maximum number of people that can actually pass through the vertex. The numerator on the side after the vertex show the final amount of flow that has passes through and the denominator shows the maximum amount of flow that can be received. The arrow pointing down, towards the next vertex, shows the amount of excess flow from the first vertex, so the numerator shows the amount of flow that has exceeded and the denominator shows the amount of flow there is in the beginning.

The value of flow is denoted by $|f|$. This represents the amount of flow passing from the source to the sink. The maximum flow problem is to maximize $|f|$, which is to route as much flow as possible from the source, S, to the sink, T. The minimum cut problem is to minimize the amount of capacity of an s-t cut.

Therefore, the max-flow min-cut theorem states: The maximum value of an s-t flow is the same as the minimum capacity of an s-t cut.

2.3.1 Optimal solutions

The max-flow min-cut theorem follows from the strong duality theorem, which states that if the primal program, a max-flow min-cut graph with one edge, has an optimal solution, x^*, then the dual program, the max-flow min-cut directed graph with two edges, also has an optimal solution, y^*, such that the optimal values formed by the two solutions are the same.

Example:

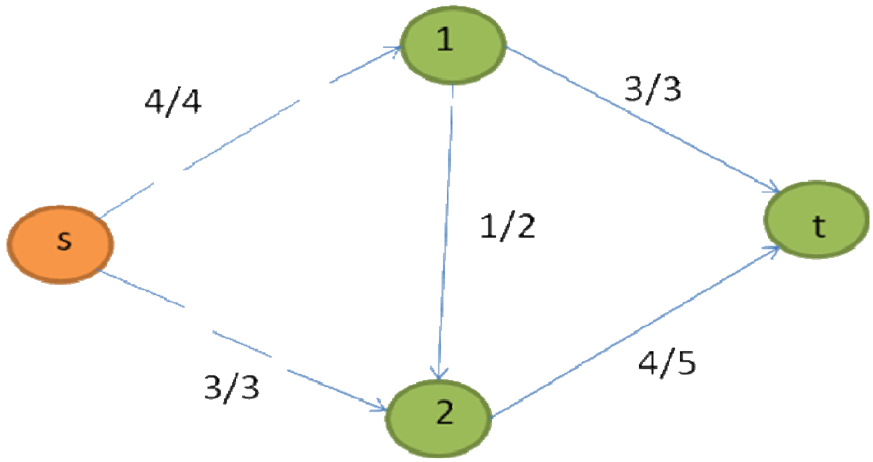


Figure 2.3.1.1

A network with the value of flow equal to the capacity of an s-t cut

The figure has a network having a value of flow of 7. The vertex in orange and the vertices in green form the subsets *S* and *T* of an s-t cut, whose cut are the dashed edges. Since the capacity of the s-t cut is 7, which is equal to the value of flow, the max-flow min-cut theorem tells us that the value of flow and the capacity of the s-t cut are both optimal in this network.

Chapter 3: Methodology

3.1 Overview

At the end of the academic year, secondary two students would choose their GCE O level examination subjects. Streaming is based on student's total average of their English, Mathematics and Science results, together with their total average followed by choice.

3.2 Outlining the System

In this project, we decided to look at the express stream only and leave the normal academic students aside first. In the express stream, each class consists of a few options to determine the elective and pure humanity subject a student would be taking. However, a single class can only hold up to 40 pupils at most. Pupils who did not get into their first choice class would have to be re- streamed.

We would be using the subject combinations of 2010 offered in our school as the subject combinations available. In our school, students in the first class (option 1 and 2) would only be opened to elective humanity and not pure humanity. Students are allowed to do music, art or media studies if they want to. However in our school these subjects are held during extra curriculum and are optional. Therefore we would not be taking these subjects into account. The table below shows the options opened for the pupils in the express stream.

Subject Combination	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
English	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Mathematics	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Mother Tongue	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
SS/ Elect Geog		*	*		*			*		*		*	*	*	*
SS/ Elect Lit	*			*		*	*		*		*				
Additional Mathematics	*	*	*	*	*	*	*	*	*	*	*	*			
Physics*	*	*	*	*	*	*									
Chemistry*	*	*	*	*	*	*	*	*	*	*					
Biology*	*	*					*	*	*	*					
Phy/Chem											*	*			
Bio/Chem													*	*	*
Pure Geography				*			*				*				
Pure Literature			*					*				*	*	*	*
Pure History					*	*			*	*					
Food & Nutrition													*		
Principles of accounts															*
Art														*	
Music															
Media Studies															
Total Number of Subjects	8	8	8	8	8	8	8	8	8	8	7	7	7	7	7

Table 3.2

Options opened for the Pupils in the Express Stream

According to the Ministry of Education in Singapore, a student is required to do social studies (SS) as one of their humanity subjects which can be either elective or pure modules, the other elective subjects that can be taken would then be an elective of either geography, literature, history. A student is also require to do at least one science subject, Chemistry, either combined or pure. Therefore, we can use sets to help us separate the options that are opened from those that are closed.

3.3 Application of Sets

Using sets, we would be able to divide the pupils into their science subjects as pupils with the same science subjects have the most number of subjects in common. This would be easier to group them into classes.

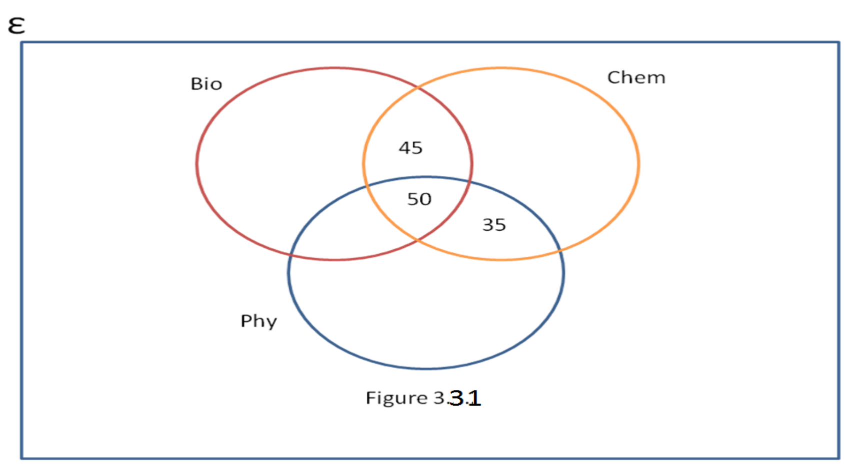


Figure 3.3.1

In Figure 3.3.1, ε represents the total number of students who want to take pure sciences. As Chemistry is a compulsory subject, no student is allowed to take only Biology and Physics. Through Figure 3.3.1, we can see that triple science is the most popular option.

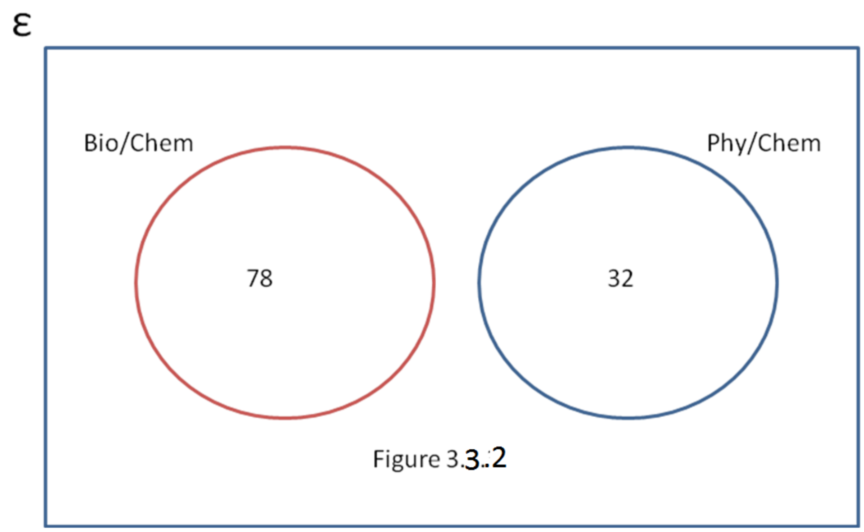


Figure 3.3.2

Figure 3.3.2 shows the pupils who want to take combine Physics or combine Biology. ε is the total number of pupils who want to take combine sciences. This figure allows us see that combined bio/ chem is the more popular choice.

With sets, we would also be able to group options together into classes. The maximum number of students needed to open a class for an option is 40 while the minimum is 5. This would allow us to see how many classes should be opened for an option and which options should be combined together into a class.

We have decided to group the student's options by their sciences, as this is the main module they would be doing, meaning that this is the module that separates the different choices the best. Below is the diagram of the options of the students grouped into the respective classes.

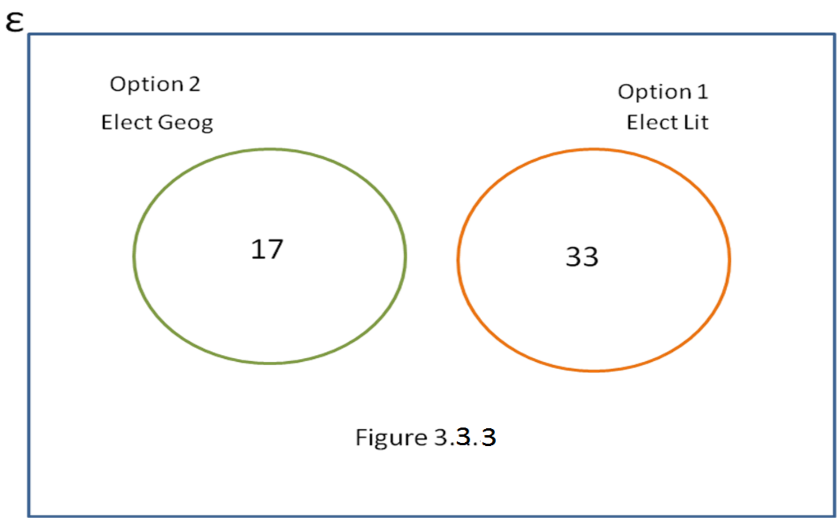


Figure 3.3.3

In Figure 3.3.3, ε represents the 50 students who have chosen triple science. Both options have sufficient pupils to be opened.

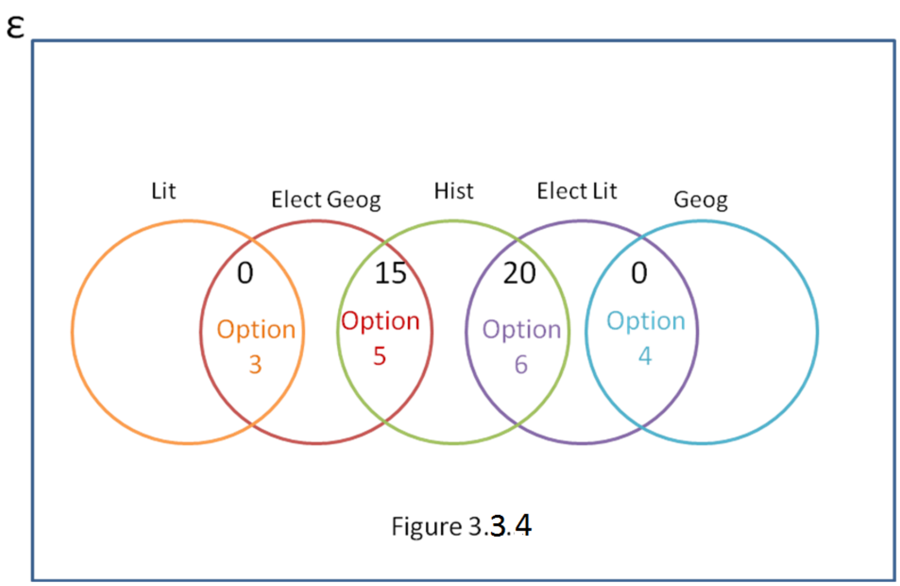
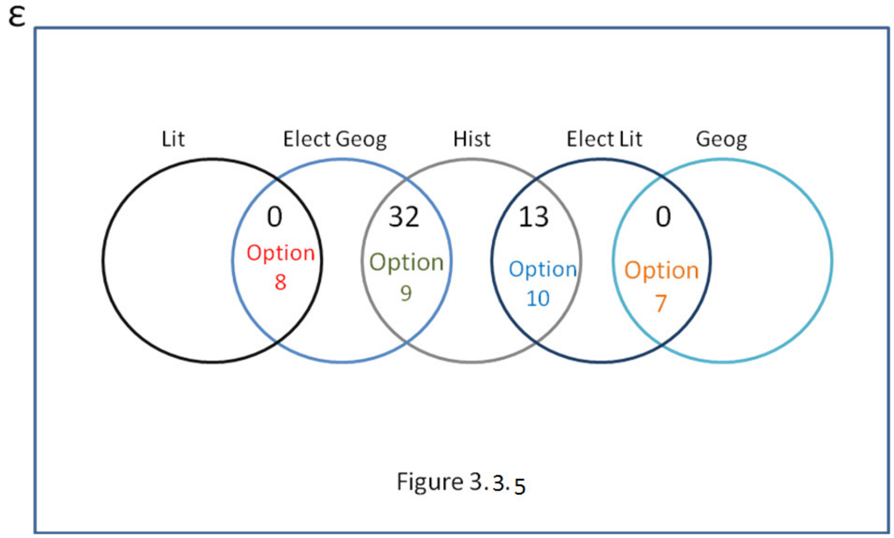
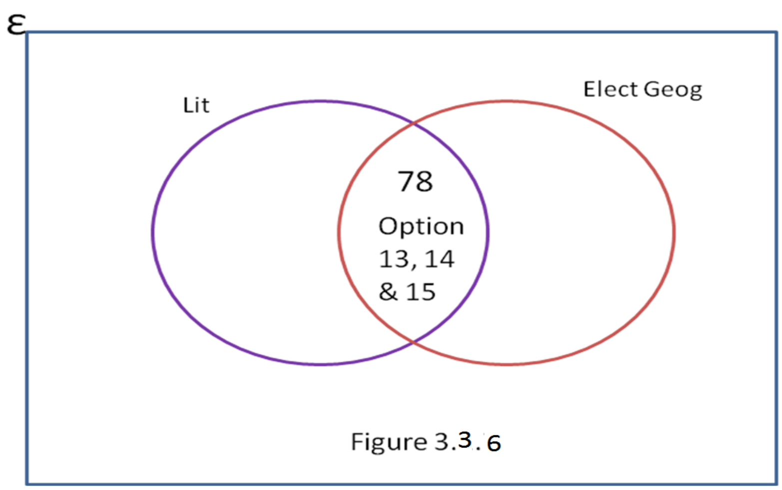


Figure 3.3.4

In Figure 3.3.4, let ε represent the 35 students that have opted to take pure Physics and Chemistry. Options 5 & 6 have enough people to be opened.



In Figure 3.3.5, ε represents all the 53 students who have opted to take pure Biology and Chemistry. Options 9 and 10 have enough people to be opened.



In Figure 3.3.6, ε represents the 78 students who have opted to take combined biology and chemistry. There are enough students for the options to be opened.

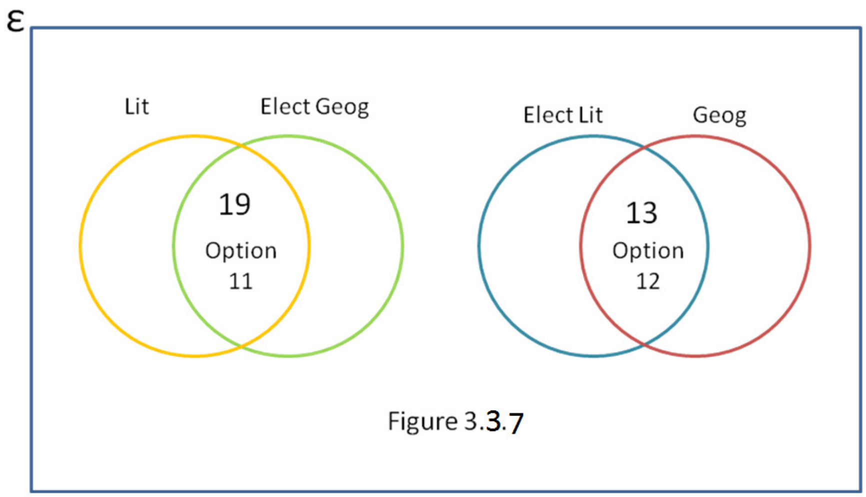


Figure 3.3.7

In Figure 3.3.7, ε represents the 32 pupils who have opted to take combined physics and chemistry. These options each represent a different class. However, due to the lack of pupils, only 1 class will be opened with both the options available in that class. The remaining pupils will sadly be placed in their next few options.

3.4 Application of Max-Flow Min-Cut

The Max-Flow Min-Cut Theorem streams the pupils into their classes. No student would be left out in the streaming. The pupils get into their streamed classes based on first merits followed by choice. Figure 3.3.1 illustrates the max-flow min-cut theorem

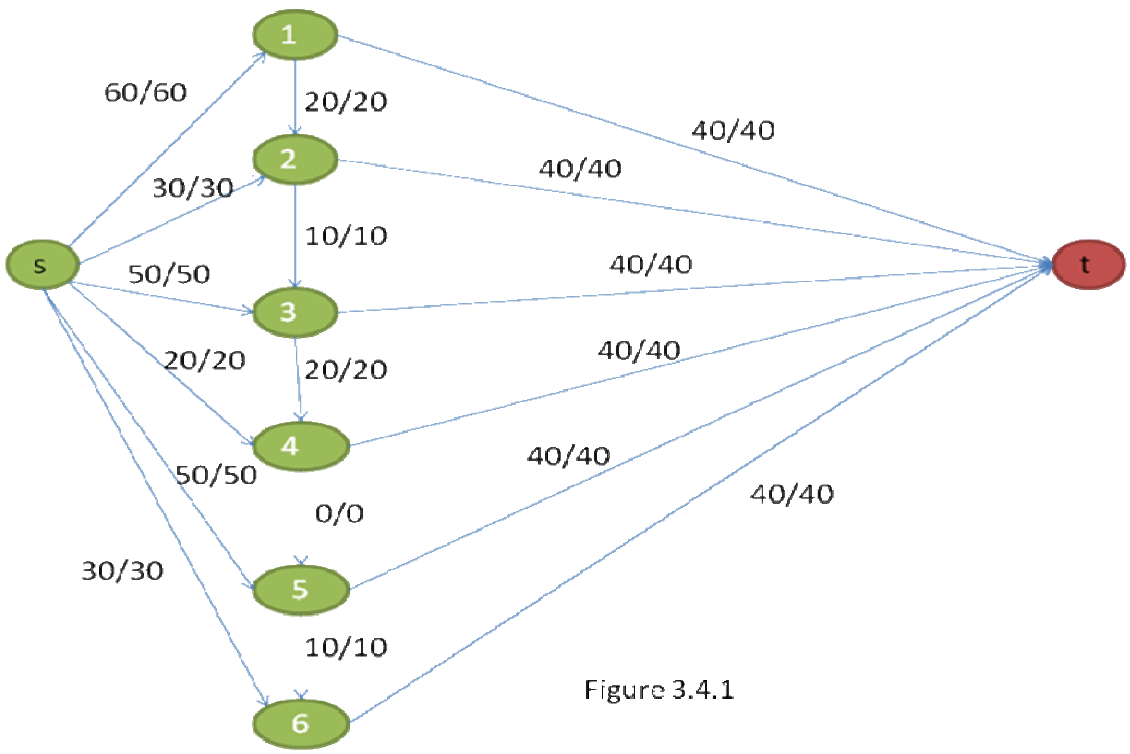


Figure 3.4.1

Each numbered bubble represents a class where s is the start point and t is the end point. Each class represents the options that have been grouped together (e.g. option 1 and option 2). The arrows represent the flow of pupils into each class. The first arrow 60/60 shows that 60 pupils want to get into class 1. However, only 40 pupils can enter a class and therefore, the other 20 pupils will go into the next class, assuming they all want to enter the classes based on their options. However, we will be making some changes in the method as we will not be able to use the method on the real streaming itself. Therefore, we changed the way the flow flows. Instead of filling up all the empty spaces, the flow will just move to the bottom of the max flow min cut diagram and to the endpoint.

The following table would show which class consists of what options and what the bubbles represent.

Classes	Option
Class 1	1,2
Class 2	3,4,5,6
Class 3	7,8,9,10
Class 4	11,12
Class 5	13,14,15
Class 6	13,14,15
XXX	People who failed to get 1st option

Table 3.4.1

List of class names and options

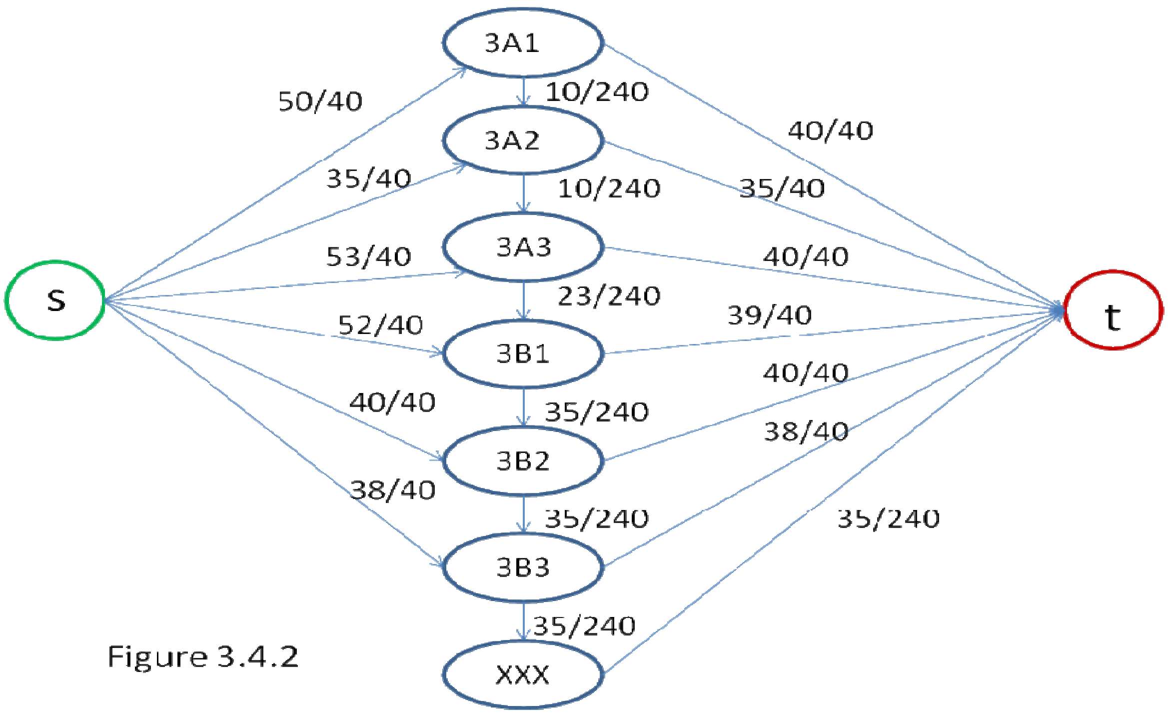


Figure 3.4.2

Each numbered bubble represents a class where s is the start point and t is the end point. Each class represents the options that have been grouped together (e.g. option 1 and option 2). The arrows represent the flow of pupils into each class. The first arrow 50/40 shows that 50 pupils want to get into class 1. Only 40 pupils can enter a class and therefore, the other 10 pupils will go flow on to the second class, class 2. However, these pupils will not be entered into class 2. The diagram shows class 2 having only 35 pupils wanting to enter, thus there are no extra pupils, so only the excess of 10 pupils from class 1 will advance to the next class, class 3. Having 45 pupils wanting to join class 3, there is an excess of 5 pupils, thus 23 pupils will flow on to the next class, this will continue and all these excess pupils will move to the bottom of the diagram and then to the endpoint. From then on, the excess pupils will be arranged according to their second choice and the max-flow min-cut theorem be used again with the denominator of each flow being the extra spaces in the each class.

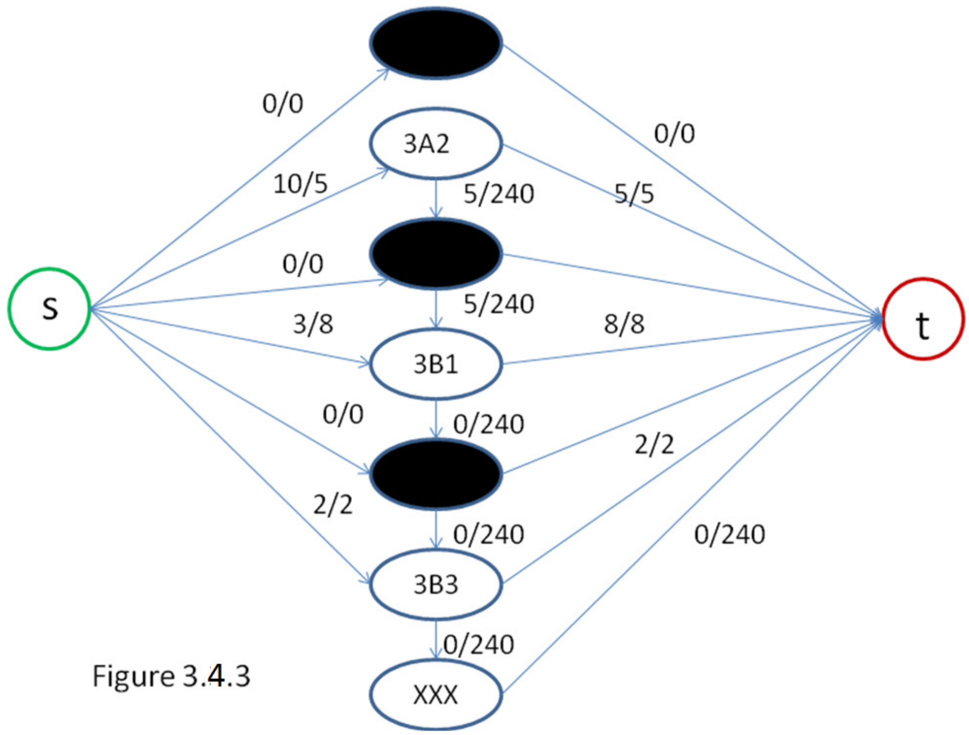


Figure 3.4.3

This is the second round of streaming for the pupils who did not make it into their 1st option. From here on, streaming is based on choice, not merit. The bubbles in black are those that have no available space left while the numbers on the vertices represent the flow of pupils. The denominator of the number being the amount of space available and the numerator being the number of pupils who want to enter the class.

As this is the second and also the final round of the streaming for our case, the pupils who did not get into their subsequent option would be placed in a class with available spaces.

3.5 The Streaming Process

We would first use sets to find the options that are opened and those that are closed. Option 1 and option 2 would be grouped together as they have the most number of subjects in common. This follows to the other options. This would allow us to decide which options should be grouped together into the same class.

Using Microsoft excel, we would then key in the data. Next, we would arrange the pupils, first by option and then by average. This can be done in Microsoft excel by firstly clicking the 'Home' button and next the 'sort and filter' button. Then after clicking the 'custom sort' button, we would type 'option' into the box next to 'sort by', then, we click the 'add level' button and type 'average' into the box next to 'then by'. We then click the 'ok' button.

After doing this, we group the students that options join to be the same class. For example, option one and option two. Then we would re-rank them among their class. We would next take the first forty pupils of the class or everyone if there are less than forty pupils. However, if there are less than 5 pupils in an option the option would be closed and the pupils would have to be streamed again to their subsequent options. The pupils that were not yet placed into a class would then be ranked again according to their average. They would be given their second choice. If that class has no extra space, the pupils would then get their next option. This will continue until all the pupils are placed into their classes.

Chapter 4: Analysis

4.1 Advantages

The advantage of our method is that by making use the Microsoft Excel program that is widely available and easily installed into a computer, it allows the streaming process to be slightly more convenient, not needing some specialized program to run the data. Using the Microsoft Excel program, data can be easily sorted and organized.

4.2 Disadvantages

Our method is fairly accurate, however it still requires a person to go through the data. Human error may occur at one time or another due to the many hours trying to sort pupils into suitable classes, causing some pupils to be missed out. This may occur especially when re-streaming pupils into their second choice. As there would be many more pupils to stream into their first option, it would be easy to forget about the pupils that have yet to be re-streamed. We have tried highlighting the pupils that have to be re-streamed into their second option, however there would still be instances where there would be some pupils missed out.

Furthermore, when we re-streamed the students the second time, we streamed them based on choice and not by merit. If the student is not satisfied with their allocated combination of subjects, they would have to appeal to the school for a change in subject combination. It is then up to the school to decide if the student can be transferred into that class. Even if the class already has forty pupils, exceptions could be made with the allowance of one or two more pupils. This breaks our criteria of forty pupils in a class.

4.3 Limitations

This method we have come up with only takes the humanities and science subjects into account and not extra subjects such as food and nutrition and principle of accounts. We also have not modified this method as a general method for all schools as they may offer different subjects that may require different grouping such as the inclusion of design and technology and other subjects.

Furthermore, we did not take into consideration the number of available classrooms for the secondary three cohort as this might limit the number of options that can be opened even if there are enough people.

Chapter 5: Conclusion

As we were not able to compare our method with the currently existing method, we were unable to come to a conclusion as to whether our new method was more effective or efficient. All in all, this method we have arrived at is fairly accurate and is able to stream students quickly. However it does have its advantages and disadvantages, this method has proven quite effective in streaming our mock pupils into their desired classes with most of the pupils getting into their first option.

Further extensions could be made to this project by extending this method to include other subjects offered by other schools such as design and technology. This would make it a general method that all schools may choose to use for their streaming process.