FACIAL RECOGNITION

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An essay presented to Singapore Mathematical Society Essay Competition

2010

Abstract

Social science refers to the area of studies that explore human society. Some examples of social sciences are geography and linguistics. In this essay, we focus on anthropology, and the use of this study in forensics, an area of social science. Mathematics is involved in this area of social science as anthropologists seeking to identify human remains require specific calculations to determine characteristics, such as the race, gender, age and stature. Our essay focuses on three-dimensional facial reconstruction, a technique that requires the use the Principal Component Analysis, an in-depth mathematical calculation. The relationship between mathematics and this area of social science is investigated in the essay.

1. Introduction

Social science is refers to the area of studies that explore human society. Some examples of social sciences are anthropology, psychology, political science, geography and linguistics. Mathematics is applied in many areas of social science. For example, mathematics is used in psychology to quantify and analyse scientific results in an organized method through statistics. Factor analysis of probability density functions and sampling distributions are common statistical concepts that are used in psychology. The Stevens' power law is an application of mathematics to show the relationship between magnitude of physical change in the environment and the intensity that humans feel from it. Applications of mathematics can also be found in political science. For example, mathematical tools are used to study voting systems. Economist Kenneth Arrow used mathematical methods to seek out the criteria of a "fair" voting system and derived Arrow's Impossible Theorem. Mathematical concepts are also applied to analyse the benefits and consequences of voting systems. Anthropology also requires the applications of mathematics, and is our focus for this essay.

1.1 Anthropology

A quote from Oxford Dictionary states that "Anthropology is the study of human origins, societies and cultures," There are many uses for mathematics in anthropology. For example, mathematics is applied in forensic anthropology to specifically calculate the certain characteristics of an unidentified body, such as its stature and face shape. Forensic anthropology is an application of scientific methods and the study of human origins for identification of human remains and crime investigation. Forensic anthropologists are able to identify the gender, age, stature and the ancestry of the body just by studying the bones of the remains. Forensic anthropology is very important as it enables unknown individuals to be identified and their cause of death detected. This is essential especially in situations of mass fatalities.

2. Mathematics Applications in Forensic Anthropology

In the second chapter of our essay, mathematics applications in forensic anthropology, such as the calculation of stature and the use of Principal Component Analysis (PCA) in facial recognition, are explained. Eigenfaces, eigenvectors and eigenvalues which are required in Principal Component Analysis (PCA), are also described in this section of our essay.

2.1 Stature

One of the processes of identifying human remains in forensic anthropology is in the aspect of stature, or height. Stature can be calculated through the "Trotter – Gleser Formula", created in 1952 by Mildred Trotter and Goldine C. Gleser. This formula is used for the calculation of estimated stature from human long bones, and was derived based on a population of American casualties of the Korean War and the Terry collection of human remains, a collection of 1,728 human skeletons held by the Department of Anthropology of the National Museum of Natural History of the Smithsonian Institution in America. The height of a person can be calculated through this formula just by a measurement of specific bones. For example, if a forensic anthropologist only had the humerus (upper arm bone) of a body, he or she could use the length of this to estimate the height of the person when he or she was alive. Below is the formula for this estimation:

$$1.26 \times (\text{length of humerus}) + 62.10 = \text{stature } (+/- 4.43 \text{ cm})$$

For example, if Woman A's humerus was 25 centimetres long, her height would be:

3.26 x 25 +62.10 = 143.6cm (therefore height is anywhere in between 139.17cm to 148.03cm) This calculation can also be done with different sets of numbers for different bones.

This is but one example of how mathematics aids the work of forensic anthropologists. There are many different ways in which mathematics can be used in the works of forensic anthropologists.

2.2 Mathematics and Facial Reconstruction

To assist in the identification of the person for crime investigation, forensic anthropologists can sometimes be asked to recreate the face of the skeleton by facial reconstruction. In three-dimensional facial reconstruction, computer programs used to give anthropologists an idea of what the person looked like in life. Our essay will focuses on this aspect of mathematics in forensic anthropology.

Facial reconstruction consists of two-dimensional reconstruction and three-dimensional reconstruction. Two-dimensional reconstruction requires skull and ante mortem (near the time of death) photographs, while three-dimensional facial reconstructions include the use of clay sculptures; and the use of high resolution three dimensional computer illustrations.

2.2.1 Three-dimensional Facial Reconstruction

In the past, three-dimensional facial reconstruction can only be carried out manually by clay sculpture. Forensic artists need to know the depth of skin which covered the skull. They usually scatter 20 to 35 tissue layers over the face. Heavily concentrated depths are mainly around the mouth and between the eyes. Tissue depth markers are applied in the proper pre-determined location to mark out how deep the person's flesh had been when he or she was alive. Pieces of clay that have been made to match the height of the tissue depth markers are then placed between them and once the strips are in place, clay is used to fill the gaps between each tissue depth markers. This forms the clay sculpture of the face and helps anthropologists know what the victim looked like when they were alive. This manual process is a long and tedious one, performed by skilled forensic artists. In the past, this was the only method of three-dimensional facial reconstruction and required much time and effort.

Due to technological advances in recent years, three-dimensional facial reconstruction can now be performed on computer programmes. Using the tissue depth markers as guidelines, a computer programme is able to cover the skull with computer generated muscles. These computer programmes make use of mathematical methods to derive the image. By applying Principal Component Analysis (PCA), geometry and eigenfaces, estimations of the unidentified face are ensemble to form the main features of the face. A set of eigenfaces is generated by PCA, a mathematical process. Mathematical knowledge of matrixes and vectors are also applied in PCA.

2.2.2 Eigenfaces, Eigenvectors and Eigenvalues

In mathematics, eigenvectors and eigenvalues are related terms in the area of linear algebra. The prefix "Eigen" means "own" or "idiosyncratic" in the German language. Eigenvalues and eigenvectors are properties of a matrix. Eigenfaces are a set of eigenvectors used in the computer vision (i.e. involves the processing of real-world images by machines) problem of facial recognition. Eigenfaces can be used for facial recognition. This method was developed by Sirovich and Kirby in 1987 and applied into face recognition by Matthew Turk and Alex Pentland. It considered one of the first successful attempts at facial recognition technology. Eigenvectors are vectors which, when acted on by a particular linear transformation, produce a scalar multiple of the original vector. This scale is called the eigenvalue which corresponds to this eigenvector.

2.2.3 Principal Component Analysis (PCA)

Eigenfaces, eigenvectors and eigenvalues described in the previous section, are frequently used in Principal Component Analysis. Principal Component Analysis, or PCA, is a statistical tool which expresses 1-D vectors from 2-D images into compact principal components of a feature space. The main features of the face can be reconstructed by performing PCA on an unidentified skull. However, before PCA can be performed, the skull needs to be placed on a workable stand where it can easily be tilted and turned in all directions. The skull is then positioned in the Frankfort Horizontal position for scanning. The Frankfort Horizontal position is a craniometrical plane determined by the inferior borders of the eye socket and the upper margin of the internal auditory canal. The scanning allows vertices to be formed on the skull and this data is then recorded on a computer. After positioning the skull, the collected data from the scan is then adjusted by subtracting the mean of each data dimensions. The mean of x is subtracted from the values of x, mean of y is subtracted from the values of y, and mean of z is subtracted from the values of z. As the skull is three-dimensional, the vertices of the skull will be presented as (x, y, z). To calculate the final data, covariance matrix of (x, y, z) must be derived.

$$C = \begin{pmatrix} cov(x,x) & cov(x,y) & cov(x,z) \\ cov(y,x) & cov(y,y) & cov(y,z) \\ cov(z,x) & cov(z,y) & cov(z,z) \end{pmatrix}$$

$$Cov(x,y) = \sum \left\{ [x - \sum x][y - \sum y] \right\},$$

where C represents the covariance matrix

A covariance matrix is a statistical measure of the variance (measure of spread of scores) of two random variables that are observed or measured in the same mean time period. This measure is equal to the product of the deviations of corresponding values of the two variables from their respective means.

After deriving the covariance matrix, its eigenvalues and eigenvectors are calculated to obtain the final data. The principle component which is the eigenvector with the largest eigenvalue is chosen to form a feature vector. Lastly, to obtain the final data, the row feature vector is multiplied with the adjusted data.

$$FinalData = RowFeatureVectors \times RowAdjustedData$$

The final data is the estimation the face of the identified skull.

3. Conclusion

Facial reconstruction is one of the numerous ways mathematics is made use of in anthropology. Three-dimensional facial reconstruction using computers has been made possible due to the use of mathematics in anthropology. Before this technological advancement, all three-dimensional reconstruction required the clay sculpting method. With the use of mathematics in a computer programme, the process of facial reconstruction can be done much quicker than before. This enables forensic anthropologists to identify the deceased quickly when all that remains is his or her bones. Principal Component Analysis used in three-dimensional facial reconstruction is being described and explained in our essay. Applications of eigenfaces, eigenvectors and eigenvalues in Principal Component Analysis are also introduced. Facial reconstruction is but one of the numerous ways forensic anthropology is aided by mathematics. In all, there are indeed many connections between the two areas of study.

BIBLIOGRAPHY

Forensic Anthropology, 8th June 2010, <http://www.forensic-medecine.info/forensic-anthropology.html>

K. Miller, Forensic Science Communication, Survey of Tissue-Depth Landmark for Facial Approximation, January 2004, 8th June 2010, <http://www.fbi.gov/hq/lab/fsc/backissu/jan2004/research/2004_01_research02.htm>

Facial Reconstruction, Thinkquest Team, 2005, 8th June 2010, < http://library.thinkquest.org/04oct/00206/lo_pti_facial_reconstruction.htm >

World of Forensic Science, Facial Recognition, 2010, 8th June 2010, <http://www.enotes.com/forensic-science/facial-recognition>

Facial Reconstruction, 8th June 2010, <http://www.forensicartist.com/reconstruction.html>
Lindsay I Smith, A tutorial on Principal Component Analysis, 26th February 2002, 8th June 2010, <http://www.cs.otago.ac.nz/cosc453/student_tutorials/principal_components.pdf>

Wesley Turner, Computer-Aided Forensics: Facial Reconstruction, 9th June 2010, < http://iospress.metapress.com/content/ey4e056uljfl4h83/>

Variance, Answer.com, 9th June 2010, <http://www.answers.com/topic/variance>

What is a variance covariance matrix, Answer.com, 9th June 2010, <http://wiki.answers.com/Q/What_is_a_variance_covariance_matrix>

Covariance, Answer.com, 8th June 2010, <http://www.answers.com/topic/covariance>

Eigenvalue, eigenvector, eigenspace, Answer.com, 9th June 2010, <http://www.answers.com/topic/eigenvalue-eigenvector-and-eigenspace#Technical_definition>

Covariance, NetMBA Statistics, 9th June 2010, <http://www.netmba.com/statistics/covariance/>

Eigenvector Definition, Learnthat, 1st July 2010, < http://www.learnthat.com/define/view.asp?id=4185>

Stevens' Power Law, Wikipedia, 1st July 2010 <http://simple.wikipedia.org/wiki/Stevens'_power_law>

Mathematics and Psychology, Bookrags, 1st July 2010
<http://www.bookrags.com/research/mathematics-and-psychology-wom/>

Mathematics and Social Sciences, Dartmouth College, 1sy July 2010
<http://www.dartmouth.edu/~reg/courses/desc/mss.html>

Plane, The Free Dictionary, 1st July 2010
<http://medical-dictionary.thefreedictionary.com/plane>